CANADIAN JOURNAL OF PHILOSOPHY
Volume 37, Number 2, June 2007, pp. 245-262

Material Points and Formal Concepts in the Early Wittgenstein

ANDREAS BLANK
The Cohn Institute
Tel Aviv University
Ramat Aviv
69978 Tel Aviv
Israel

I Introduction

In an influential article, Gerd Grasshoff has argued for the identification of the objects in Wittgenstein's *Tractatus*[1] with the ultimate constituents of reality in Heinrich Hertz's *Principles of Mechanics*. Grasshoff's interpretation is based on two interrelated claims: (1) The specific determination of the objects in the world and the relation among them is the primary theme in Wittgenstein's early philosophy, *because* it is the primary theme for Hertz.[2] (2) Wittgenstein did not assume the existence of

1 Abbreviations: Ludwig Wittgenstein, *Notebooks 1914-16*, 2nd ed. G.H. von Wright and G.E.M. Anscombe, eds., G.E.M. Anscombe, trans. (Oxford: Blackwell 1979) [= NB]; *Prototractatus: An Early Version of Tractatus Logico-Philosophicus*, B.F. McGuinness, T. Nyberg, G.H. von Wright, eds., D.F. Pears and B.F. McGuinness, trans. (London: Routledge & Kegan Paul 1971) [= PT]; Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, D.F. Pears and B.F. McGuinness, trans. (London: Routledge and Kegan Paul 1971) [= TLP]; Ludwig Wittgenstein, *Culture and Value*, G.H. von Wright, ed., P. Winch, trans. (Chicago: University of Chicago Press 1980) [= CV]; Heinrich Hertz, *The Principles of Mechanics*, D.M. Jones and J.T. Wiley, trans. (Boston: Dover 1956) [= PM].

2 Gerd Grasshoff, 'Hertzian Objects in Wittgenstein's *Tractatus*,' *British Journal for the History of Philosophy* **5** (1997), 95-102. A similar interpretation has been suggested by

simple objects on purely logical grounds without having specific examples of simple objects in mind, *because* Hertz did not do this.[3] The first of these claims tries to remedy what Grasshoff calls the 'thematic' misunderstanding of the *Tractatus*, the second what he calls the 'elementary' (or 'logicistic') misunderstanding, and both together are intended to revise what he calls the 'biographical' misunderstanding that ascribes to Frege and Russell a more significant influence on the early Wittgenstein than to Hertz.[4]

Indeed, to interpret Tractarian objects as Hertzian objects is suggested by TLP 6.3431, where Wittgenstein says that the laws of physics 'indirectly speak about the objects of the world.' Moreover, in TLP 6.3432, Wittgenstein adds that mechanics mentions not '*particular* material points' but '*some points or other*.'[5] This suggests that material points — one kind of the entities that play a fundamental role in Hertz's mechanics — belong to the simple objects in the technical sense of the *Tractatus*. Obviously, Wittgenstein is little concerned with the details of the technical apparatus of Hertz's *Principles of Mechanics*. As David Keyt has pointed out, material points, for Hertz, display internal complexity and, hence, according to the terminology of the *Tractatus*, are states of affairs rather than objects.[6] Nevertheless, as Grasshoff has argued, another kind of entities in Hertz's mechanics, the so-called 'mass particles' [*Massenteilchen*], do not suffer from this difficulty.

Does the obvious affinity between Hertz's and Wittgenstein's ultimate constituents of reality imply that, for Wittgenstein, the ontology of Hertzian objects has an epistemological priority over his logic? I agree that, contrary to logicistic interpretations, Wittgenstein's early views on simple objects and their concatenations should not be seen as derived

---

James Griffin, *Wittgenstein's Logical Atomism* (Oxford: Oxford University Press 1965), 4-5; 49-50. For a critical overview of interpretations inspired by Grasshoff's article, see Alfred Nordmann, 'Another New Wittgenstein: The Scientific and Engineering Background of the *Tractatus*,' *Perspectives on Science* **10** (2002) 356-84. Grasshoff has pointed out several terminological problems in the English translation of the *Principles of Mechanics* as well as in the McGuinness-Pears translation of the *Tractatus*. Throughout the present paper, Grasshoff's suggestions concerning terminology have been adopted.

3 Grasshoff, 'Hertzian Objects in Wittgenstein's *Tractatus*,' 102-12.

4 Ibid., 88.

5 C.K. Ogden's translation; see Ludwig Wittgenstein, *Tractatus Logico-Philosophicus* (London, 1922). The emphases in this and all other citations are Wittgenstein's.

6 David Keyt, 'A New Interpretation of the *Tractatus* Examined,' *Philosophical Review* **74** (1965) 229-39.

from his views on names and their combinations into propositions. In this sense, there is no epistemological priority of logic over ontology.[7] However, contrary to the physicalistic interpretation, neither does the ontology of Hertzian objects have a priority over logic for the early Wittgenstein. Wittgenstein's early views on names and their combinations should not be seen as derived from his views on simple objects and states of affairs. Rather, the conceptual structure of Wittgenstein's early philosophy should be seen as a net of mutually interdependent concepts, in which neither logical nor ontological concepts can be defined independently of each other. As I argue, this conceptual structure is captured by Wittgenstein's characterization of the concept of object as a 'formal concept'. Moreover, I claim that only by taking into account this conceptual structure can we be led to an understanding of the analogy Wittgenstein sees between his own view of the method of logic and the method of Hertz's mechanics.

II Hertz on 'Pseudo-Pictures'

Wittgenstein's earliest remarks on Hertz do not directly concern two issues that have received wide attention in the recent literature, viz. the influence of Hertz's idea of scientific sentences as pictures on Wittgenstein's picture theory of language,[8] and the influence of Hertz's conception of clarification as eliminating 'idling wheels' from language on Wittgenstein's view of elucidation.[9] Rather, they address the analogy between the role of construction in logic and the role Hertz ascribes to construction in mechanics. In an entry in the *Notebooks 1914-1916*, after discussing the analogy between the way alternative physical theories lead to different descriptions of facts and the way geometrical nets with meshes of alternative (square, triangular, hexagonal) form lead to differ-

---

7 Contrary, e.g., to the view of Peter Carruthers. Cf. his *The Metaphysics of the* Tractatus (Cambridge: Cambridge University Press 1990), ch 3.

8 Peter Barker, 'Hertz and Wittgenstein,' *Studies in the History and Philosophy of Science* **11** (1980) 246-9; Andrew D. Wilson, 'Hertz, Boltzmann and Wittgenstein Reconsidered,' *Studies in the History and Philosophy of Science* **20** (1989) 258-62; Rom Harré, 'Wittgenstein: Science and Religion,' *Philosophy* **76** (2001) 214-17, David Hyder, *The Mechanics of Meaning: Propositional Content and the Logical Space of Wittgenstein's* Tractatus (Berlin and New York: De Gruyter 2002), ch. 6.

9 See Barker, 'Hertz and Wittgenstein,' 250-6; Allan Janik, 'How did Hertz Influence Wittgenstein's Philosophical Development?' *Grazer Philosophische Studien* **49** (1994/95), 21-3.

ent descriptions of patches on a surface, Wittgenstein turns his attention to the relation of logic to the method of mechanics, and writes:

> Mechanics is *one* attempt to construct all the propositions that we need for the description of the world according to a *single* plan. (Hertz's invisible masses.) Hertz's invisible masses are admittedly pseudo-objects. (NB, 35-6)

In fact, on Wittgenstein's part this may be a partial misrepresentation of Hertz's views, at least if the term 'pseudo-objects' is read as ascribing a non-real character to invisible masses. Moreover, if read from this perspective, the cited passage does not adequately represent the views expressed in the later parts of the *Notebooks 1914-1916*, which concern the role of combinations of simple objects as conditions for sense. Nevertheless, even if the remark is partially misleading, it contains some remarkable insight. In particular, it draws attention to the fact that in two prominent passages of the *Principles of Mechanics*, scientific propositions are characterized as 'pseudo-pictures' [*Scheinbilder*].[10] This fact is obscured by the English translation, which renders both occurrences — in analogy to the terminology of 'picture' used elsewhere in Hertz's work — simply as 'pictures.' These scattered occurrences of the term 'pseudo-picture' make a puzzling impression because they seem to contradict Hertz's view that scientific theories, in some sense, *are* pictures of reality. It is plausible to read the passage from the *Notebooks 1914-1916* cited above as a response to the occurrence of 'pseudo-picture' in the *Principles of Mechanics*. The first passage in which Hertz uses this term is from the Introduction to the *Principles of Mechanics*:

> We make for ourselves internal pseudo-pictures or symbols of the external objects, and we make them in a way that the necessary consequents in thought of the pictures are always the pictures of the necessary consequents in nature of the objects pictured. In order that this requirement may be satisfied, there must be a certain conformity between nature and our thought.... When from our accumulated previous experience we have once succeeded in deducing pictures of the desired nature, we can then in a short time develop by means of them, as by means of models, the consequences which in the external world only arise in a comparatively long time, or as the result of our own intervention.... The pictures which we here speak of are our conceptions of things. With the things themselves they are in conformity in *one* important respect, namely, in satisfying the above-mentioned requirement. (PM, 1)

---

10 Cf. the original edition: Heinrich Hertz, *Die Prinzipien der Mechanik: In neuem Zusammenhange dargestellt*, *Gesammelte Werke*, vol. 3, Philip Lenard, ed. (Leipzig, 1894).

The second passage occurs at a place of comparable strategic weight at the very beginning of the second part of the book, where Hertz characterizes laws of correspondence as something by means of which the signs 'time,' 'space,' and 'mass' become parts of our 'pseudo-pictures' of external objects (PM, 302).

Hertz's terminology suggests that scientific propositions, in some respect, are pictures and, in some other respect, pseudo-pictures. Let us first see in which sense they are pictures. This sense becomes especially clear in Hertz's discussion of adequacy conditions for scientific theories:

> Different pictures of the same objects are possible and these pictures may differ in various respects. We should at once denote as inadmissible all pictures that implicitly contradict the laws of thought. Hence we demand in the first place that all our pictures shall be logically permissible.... We shall denote as incorrect any permissible pictures if their essential relations contradict the relations of external things.... Hence we demand that all of our pictures be correct. But two permissible and correct pictures of the same external object may yet differ with respect to appropriateness. Of two pictures of the same object, that one is the more appropriate which pictures more of the essential relations of the object — the one that we may call the more distinct. Of two pictures of equal distinctness, the more appropriate is the one that contains, in addition to the essential characteristics, the smaller number of superfluous or empty relations, i.e. the simpler of the two. Empty relations cannot be avoided altogether; they enter into the pictures because they are just pictures, and indeed pictures of our particular mind and, therefore, necessarily partly determined by the properties of its mode of picturing. (PM, 2)

Allan Janik understands the last criterion (the criterion of appropriateness) as a criterion of communicative effectiveness. According to his reading, theories must be constructed 'with a view to the communication situation in which the scientist finds himself.' As Janik suggests, this requirement entails in Hertz's view that theories must be formulated in such a way as to be able 'to convey the sort of information that the audience wants to learn about in a form it can assimilate.'[11] However, this reading is problematic. Certainly, Hertz does not connect appropriateness, as he does in the case of permissibility, with the nature of the human mind (see PM, 3). Nor does he associate appropriateness, as he does in the case of correctness, with the state of our present experience (see PM, 3). Rather, he characterizes it as something belonging to the level of the arbitrary choice of the means of the scientific representation of pictures, such as notations, definitions, and abbreviations — as he puts it: 'in short, all that we can arbitrarily add or take away.' Moreover, he claims, 'We cannot decide without ambiguity whether a picture is ap-

11 Janik, 'How Did Hertz Influence Wittgenstein's Philosophical Development?' 27

propriate or not.... One image may be more suitable for one purpose, another for another' (PM, 2-3). Nevertheless, the purposes mentioned by Hertz should not be understood as practical purposes. In particular, he does not regard appropriateness as a pragmatic criterion in the sense of communicative effectiveness. Rather, in his view it is something that depends on two objective factors: relational properties of objects and relational properties of pictures. As he points out, of two rival theories, the one is more adequate that maximises essential relations and minimises inessential ones (PM, 2). For example, a system that excludes all or at least a part of non-natural movements would 'mirror more real relations between objects and thus be in this sense more appropriate' (PM, 11). Similarly, an appropriate system of representation must include 'the whole multiplicity of rigid connections which may arise between the bodies of nature' (PM, 19). In this sense, he claims, 'We are justified in deciding that if our images are well adapted to the things, the actual relations of the things must be represented by simple relations between the pictures' (PM, 23). Consequently, the arbitrariness of the tools of representation has to do with the fact that each mode of representation includes, in addition to essential relations, inessential relations belonging only to the system of signs itself. Thus the different purposes a given system of representation can fulfil, in addition to pragmatic purposes, are different ways of combining the representation of particular essential and inessential relations. As to the representation of a particular essential relation, one mode of representation may be more appropriate than another, while as to the representation of another essential relation a different mode of representation may be more appropriate. Hertz emphasizes that he is not talking about adequacy in terms of communicative appropriateness. The adequacy he has been talking about is appropriateness 'in the sense of a mind which independently of the contingent position of man in nature strives to comprise the totality of physical knowledge in an objective way' (PM, 40). He compares the relation between his exposition of mechanics and classical mechanics to the relation between a systematic grammar of a language and a grammar that allows the student of the language to acquire the basic communicative skills in a short time (PM, 40). Thus it is classical mechanics, not Hertz's new exposition of the principles of mechanics, that is shaped in such a way as to meet communicative needs. Adequacy, however, means something different for Hertz, namely the capacity of scientific theories to mirror by means of the relations between the signs constituting them the real relations between objects. In this sense, scientific theories are representations of the pictures we make for ourselves of things.

At the same time, scientific theories are representations of pseudo-pictures because they are non-committal regarding the qualitative side of the ultimate constituents of reality. As Hertz points out, the definitions

of the first book of the *Principles of Mechanics* are a priori judgements that 'are based on the laws of inner intuition and the forms of the specific logic of the person making judgements and have no other connection with experience than these intuitions and forms' (PM, § 1; see § 2; § 295; § 296). Again, at the very end of the *Principles of Mechanics*, the definitions introduced in the first book are characterized as describing 'necessary relations between the productions of our own mind' (PM, 734). Thus, the necessity involved on the side of a system of signs is one relative to a set of laws of inner intuition and arbitrary decisions concerning the instruments of representation. The definitions of concepts such as 'hidden mass' and 'mass particle,' therefore, are part of set of 'productions of our own mind' related to each other by relations of logical necessity. Thus, what Hertz says about hidden masses and mass particles is to be seen as something necessarily required within a certain mode of representing the external world. According to Hertz, 'if we wish to obtain an image of the universe which shall be well-rounded, complete and conformable to law, we have to presuppose, behind the things which we see, other, invisible things' (PM, 25). In particular, his suggestion to eliminate concepts such as force or energy from the independent fundamental conceptions of mechanics requires adding a hypothesis that gives mechanics the same logical multiplicity, which the facts it describes have. According to Hertz, 'the deficiency in the multiplicity which thus results in the fundamental conceptions necessarily requires some complement' which is to account for the fact that 'the multiplicity of the actual universe must be greater than the multiplicity of the universe which is directly revealed to us by our senses' (PM, 25). As to the entities involved in his mechanics, he holds that 'this mode of conception is just our hypothesis' (PM, 25). This is compatible with his claim to the effect that his suggested mode of representation, in contrast to theories involving the concepts of force or energy, does not refer to 'entities of a special and peculiar kind.' In his view, hidden masses are not entities that 'belong to a special category' (PM, 25). They are objects 'of the same kind as sensory experiences.' They cannot, therefore, in themselves, 'present anything mysterious to us' (PM, 28). Hidden masses, therefore, are seen as objects of the same ontological status as the masses given in experience.[12] However, according to Hertz, what the laws of mechanics describe is the behaviour of *systems* of material points. Thus, all experience is an experience of the behaviour of systems of material points. Already a single material point,

12 See Michael Heidelberger, 'From Helmholtz's Philosophy of Science to Hertz's Picture-Theory,' in *Heinrich Hertz: Classical Physicist, Modern Philosopher*, D. Baird, R.I.G. Hughes, and A. Nordmann, eds. (Dordrecht and Boston: Kluwer 1998), 21-3.

defined as 'a finite or infinitely small mass, conceived as being contained in an infinitely small space,' is seen as an *abstraction* from a system accessible in experience (PM, § 5; § 8). The mass contained in a given space, in turn, is defined as 'the number of mass particles in any space, compared with the number of mass particles in some chosen space at a fixed time' (PM, § 4). A mass particle, defined as 'a characteristic by which we associate without ambiguity a given point in space at a given time with a given point in space at any other time' (PM, § 3), is even one further layer down in the hierarchy of the constitution of the world. Thus claiming that hidden masses are 'of the same category' as visible masses is compatible with the view that mass particles are of a different category from objects accessible in experience. The simple constituents of complex objects are not masses, nor material points, but entities that are presupposed in the definition of the concepts of mass and material point. Mass particles are only characterized by means of their formal characteristics: they are the factors of particularity that allow one to associate a given point in space at a given time with another point in space at another time. In the sense that the propositions of physics do not carry any ontological commitment that goes beyond this purely formal characterization of mass particles, they are representations of what Hertz calls 'pseudo-pictures.'

III Wittgenstein on the 'Method of Mechanics'

What has been said in the previous section should make clear what motivated Wittgenstein in claiming that Hertzian objects are 'pseudo-objects' and that the characterization of objects as pseudo-objects is the outcome of the attempt to construct all the propositions that we need for the description of the world according to a single plan (see NB, 35-6). Even if Wittgenstein's reading of Hertz is not wholly accurate as an interpretation, it captures some often-neglected aspects of Hertz's methodology. Moreover, as we have seen, it underlies Wittgenstein's comparison between the 'method of mechanics' and the 'method of logic.' Indeed, similar to the way Hertz introduces hidden masses as a condition of the possibility of the propositions of physics, Wittgenstein introduces simple objects as a condition of the possibility of propositions: 'The possibility of propositions is based on the principle that objects have signs as their representatives' (TLP 4.0312). Moreover, a further condition of the possibility of propositions is that they are 'logically articulated' (TLP 4.032). Furthermore, what 'logically articulated' means is clarified by means of a comparison between propositions and the role Hertz ascribes to dynamical models:

> In a proposition, there must be exactly as many distinguishable parts as in the situation that it represents.
> The two must possess the same logical (mathematical) multiplicity. (Compare Hertz's mechanics on dynamical models.) (TLP 4.04)

Thus the existence of simple objects is seen as a condition of the possibility of propositions because only under this condition a proposition and a state of affairs can have the same logical multiplicity. The concept of formal or internal property underlies a series of entries in the *Tractatus* about internal properties and relations: 'In a certain sense we can talk about formal properties of objects and states of affairs, or, in the case of facts, about structural properties: and in the same sense about formal relations and structural relations' (TLP 4.122). In brackets, Wittgenstein adds: 'Instead of ''structural property'' I also say ''internal property''; instead of ''structural relation,'' ''internal relation.''' Moreover, he points out that asserting that internal relations hold is precluded by the limits of language: 'It is impossible, however, to assert by means of propositions that such internal properties and relations obtain: rather, this makes itself manifest in the propositions that represent the relevant states of affairs and are concerned with the relevant object.' Understanding the notion of a simple object in the context of the notion of internal relations implies that what Wittgenstein says about simple objects can only concern their structural, formal, properties, i.e., the way they can figure in complexes.

Interestingly, Wittgenstein compares such a view of the structure of language with Hertz's view of dynamical models. As Hertz defines it, a material system is a dynamical model of a second system if the connections of the first can be expressed by coordinates as follows: (1) the number of coordinates of the first system is equal to the number of the second; (2) with a suitable arrangement of the coordinates for both systems the same equations hold; and (3) by this arrangement of the coordinates, the expression for the magnitude of displacements agrees in both systems (PM, § 418). As a corollary, he adds: 'The property which one system possesses of being a model of another, is independent of the choice of the coordinates of one or the other system, although it is only clearly exhibited by a particular choice of coordinates' (PM, § 420). Moreover, he observes that 'The relation of a dynamical model to the system of which it is regarded as the model is precisely the same as the relation of the images which our mind forms of things to the things themselves.... The agreement between mind and nature may therefore be likened to the agreement between two systems which are models of one another, and we can even account for this agreement by assuming that the mind is capable of making actual dynamical models of things, and of working with them' (PM, § 428). By comparing human cognition

with the way dynamical models represent the logical multiplicity of mechanical systems, Hertz suggests that dynamical models exemplify the general structure of thought. Moreover, he connects the structure of thought with the constructive capacities of the human mind. Wittgenstein's comparison between propositions and dynamical models indicates in which sense he shares the view that dynamical models exemplify the general structure of thought.

Wittgenstein also shares some of Hertz's views about the constructive capacities of the human mind. In the *Notes on Logic*, he interprets the capacity of constructing symbols — and thus the capacity of constructing propositions with a sense — as a capacity that has to do with the nature of human beings: 'Man possesses an innate capacity for constructing symbols with which *some* sense can be expressed, without having the slightest idea what each word signifies. The best example of this is mathematics, for man has until lately used the symbols for numbers without knowing what they signify or that they signify nothing' (NB, 100). Similarly, in the *Notebooks* he writes: 'Language is a part of our organism and no less complicated than it' (NB, 48; see TLP 4.002). On a technical level, Wittgenstein describes formulating propositions as a constructive activity starting with indefinable components that constitute elementary propositions. After having identified 'a,' 'b,' and 'xRy' as the indefinables in 'aRb,' he emphasizes: 'The ways by which we introduce our indefinables must permit us to construct all propositions that have sense from these indefinables *alone*' (NB, 99).

According to Wittgenstein, constructing propositional signs involves the construction of variables that function as logical prototypes of simple objects. Interestingly, already in the *Notebooks 1914-1916* he connects the idea of material points with the concept of logical co-ordination:

> We might conceive two co-ordinates a_p and b_p as a proposition stating that the material point P is to be found in the place (ab). For this statement to be possible the co-ordinates a and b must really determine a place. For a statement to be possible the logical co-ordinates must really determine a logical place! (NB, 20-1)

This entails that there is no characterization of logical place in purely ontological terms: 'The proposition and the logical co-ordinates: that is the logical place' (NB, 31; see TLP 3.41). The mutual dependence of logical and ontological concepts becomes apparent in the way Wittgenstein connects the issue of objects with that of variables. On the one hand, the concept of simple object is connected with the idea of a notation for generalized propositions:

> It is easy to suppose that 'individual,' 'particular,' 'complex,' etc. are primitive ideas of logic.... This error presumably is to be explained by the fact that, by employment of variables instead of the generality-sign, it comes to seem as if logic dealt with

> things which have been deprived of all properties except thing-hood, and with propositions deprived of all properties except complexity. We forget that the prototypes of symbols only occur under the generality-sign, never outside it. (NB, 107)

The connection between the concept of variables functioning as prototypes of symbols and the concept of simple object leads Wittgenstein to a suggestion that, at first glance, seems to amount to a logicistic view: 'The simple thing for us is: the simplest thing that we are acquainted with. — The simplest thing which our analysis can attain — it need appear only as a prototype, as a variable in our proposition — *that* is the simple thing that we mean and look for' (NB, 47). This passage might be read as suggesting that the concept of object for Wittgenstein is a concept defined by logical concepts alone. However, in one of the subsequent entries, he gives a negative answer to the question 'is ''name'' so to speak a logical concept?' There, he writes that 'Names signalise what is common to a single form and a single content. — Only *together with* their syntactical use do they signalise *one particular* logical form' (NB, 53).

Wittgenstein's views about the construction of logical prototypes in physics form the context in which he introduces the concepts of material point and hidden mass. In the *Notebooks*, he writes: 'The division of the body into *material points*, as we have it in physics, is nothing more than analysis into *simple components*' (NB, 67). On first sight, it might look as if here he embraces a physicalistic view concerning the nature of the ultimate constituents of reality. This impression, however, is misleading. The preceding paragraphs of the same entry discuss the role of variables in physics. The general point around which Wittgenstein's thoughts turn there is the question whether we can justly apply logic 'straightaway to *ordinary propositions*' (NB, 66). Wittgenstein answers this question indirectly by pointing out that we apply mathematics '*with the greatest success, to ordinary propositions, namely to those of physics*' (NB, 66). He describes the application of mathematics to ordinary propositions as consisting of two steps: (1) the variables in physical theorems 'simply stand for numbers and nothing else'; (2) when numbers 'are applied, we come to relations, things etc.' (NB, 67). In this sense, Wittgenstein writes, 'The complete physical propositions do after all deal with things, relations and so on' (NB, 67). Thus the question at stake here is not to determine the qualitative side of simple objects. Rather, the aim is to integrate the notion of material point into a general theory of variables. Even more explicitly, the entry of the next day connects the issues of material points, variables, and logical co-ordinates:

> If a point in space does not exist, then its co-ordinates do not exist either, and if the co-ordinates exist then the point exists too. That is how it is in logic ...
> It always looks as if there were complex objects functioning as simples, and then

> also *really* simple ones, like the material points of physics, etc.
> That a name stands for a complex object can be seen from the indefiniteness of the sentence in which it occurs. This comes from the generality of such propositions. We *know* that not everything is yet determined by this proposition. For the generality notation *contains* a prototype.
> All invisible masses, etc. etc. must come under the generality notation. (NB, 68-9)

Here, the theory of invisible masses is seen from the perspective of a theory of variables. Wittgenstein argues that, due to the impossibility of developing a purely syntactical account of variables, one has to ascribe to variables the function of logical co-ordinates that guarantee the possibility of the existence of their referent. Wittgenstein explicates this in the *Tractatus* in terms of internal relations between propositions about complex entities and propositions about simple objects. In a passage from the *Tractatus*, we find that Wittgenstein integrates a slight variant of one of the sentences from the Notebooks passages just cited:

> A proposition about a complex stands in an internal relation to a proposition about a constituent of the complex.
> A complex can be given only by its description, which will be right or wrong. A proposition that mentions a complex will not be nonsensical, if the complex does not exist, but simply false.
> When a propositional element signifies a complex, this can be seen from an indeterminateness in the propositions in which it occurs. In such cases, we *know* that the proposition leaves something underdetermined. (In fact, the notation for generality *contains* a prototype.) (TLP 3.24)

This passage hints at Wittgenstein's opinion concerning the fact that invisible masses only occur in generalized propositions. It suggests that he claims we should view this fact from the perspective of a mutual dependence between the concepts of a logical prototype and of a simple object. Similarly, according to TLP 6.3432, '[w]e ought not to forget that any description of the world by means of mechanics will be of the completely general kind.' Furthermore, in a series of *Notebooks* entries, Wittgenstein connects the idea of an a priori accessible notion of simple object with the idea of the construction of logical prototypes:

> We can only foresee what we ourselves construct.
> But then where is the concept of a simple object still to be found?
> This concept does not so far come in here at all.
> We must be able to construct the simple functions because we must be able to give each sign a meaning.
> For the only sign which guarantees its meaning is function and argument. (NB, 71)

This is exactly what Wittgenstein has in mind when he compares the method of logic with the method of mechanics: 'If I can imagine a '*kind* of object' without knowing whether there are such objects, then I must

have constructed their prototype for myself. Isn't the method of mechanics based on this?' (NB, 74) In this sense, Wittgenstein compares the method of logic with a method of mechanics that can be understood as the attempt 'to construct all the propositions that we need for the description of the world according to a *single* plan' (NB, pp. 35-36; TLP 6.343).

IV Wittgenstein on Formal Concepts

Understanding the concepts of simple object and material point from the perspective of a theory of variables in the manner suggested by Wittgenstein implies that neither logical nor ontological concepts have an epistemological priority. This is why Wittgenstein counts the concept 'object' among the formal concepts (TLP 4.126-4.1272). As he points out, the obtaining of an internal property of a possible situation 'expresses itself in the proposition representing the situation, by means of an internal property of the proposition' (TLP 4.124). Moreover, the obtaining of an internal relation between possible situations 'expresses itself in language by means of an internal relation between the propositions representing them' (TLP 4.125). Correspondingly, 'when something falls under a formal concept as one of its objects, this cannot be expressed by means of a proposition. Instead, it is shown in the very sign for this object. (A name shows that it signifies an object, a sign for a number that it signifies a number, etc.)' (TLP 4.126). This view of the nature of formal concepts accounts for the way the concept of variable is connected with the concept of simple object. According to Wittgenstein, '[e]very variable is the sign for a formal concept' (TLP 4.1271). For example, the 'variable name ''x'' is the proper sign for the pseudo-concept *object*' (TLP 4.1272). In the *Prototractatus*, he writes: '''Object,'' ''complex,'' ''fact,'' ''number,'' etc., etc. are not names of concepts — as Russell believed — but variables' (PT 4.102273). This, however, is not intended as a purely syntactic characterization of the concept of object. According to the view developed in the *Prototractatus*, every variable is a propositional variable, which in turn is nothing but a specification of its values: 'To specify the values for a propositional variable is *to give the propositions* of which the variable is the representative. The specification is a description of those propositions' (PT 5.005-5.00501; see TLP 3.317).[13] As Wittgenstein explains:

---

13 On Wittgenstein's view of variables as propositional variables, see Matthias Varga von Kibéd, 'Variablen im Tractatus,' *Erkenntnis* **39** (1993) 79-100.

> We can distinguish three kinds of description: 1. direct enumeration; 2. giving a function *F*(x,y,...) whose various values are the propositions to be described; 3. giving features which characterize those propositions.
> In the first case instead of the variable, we can simply write its (constant) values.
> In the second case, the variable is a generalized proposition.
> In the third case, the values of the variable are all propositions that possess certain *formal* properties. (PT 5.00531-5.00534)

In the *Tractatus*, he takes up talk about 'features': 'An internal property of a fact can also be called a feature of that fact (in the sense in which we speak of facial features, for example)' (TLP 4.1221). Moreover, Wittgenstein understands every variable as a sign for a formal concept (TLP 4.1271). Accordingly, propositional variables are signs for formal concepts (TLP 4.127). Thus, even without representing variables standing for simple objects in the conceptual notation by means of the variable name '*x*,' constructing propositions with certain formal similarities already amounts to constructing prototypes of simple objects.

Does this imply that, for Wittgenstein, the concept of simple object is defined by means of logical concepts only, as several interpreters have suggested? According to Anscombe, the concept of object, for Wittgenstein, is a concept defined by the syntactical role of names in elementary sentences.[14] In a similar vein, Ishiguro holds the view that the concept of object for Wittgenstein is definable with the help of logical concepts alone: an object is whatever is designated by the subject term in a completely analyzed sentence.[15] Brian McGuinness proposes another logical definition of the concept of object. His definition uses Wittgenstein's definition of sameness of signs as the possibility to substitute signs in all contexts (TLP 3.341; 3.344). Since the same sign, for Wittgenstein, always designates the same object (TLP 5.553), an object, according to McGuinness, can be defined as what all signs that are mutually substitutable for each other in completely analyzed sentences refer to.[16]

To be sure, in the *Notebooks 1914-1916* Wittgenstein talks about simple objects in the way suggested by Ishiguro: 'But how am I imagining the simple? Here all I can say is always '''x' has reference''' (NB, 45). On first

---

14 G.E.M. Anscombe, *An Introduction to Wittgenstein's Tractatus*, 2nd ed. (New York: Harper 1965), 82, 99, 123.

15 Hidé Ishiguro, 'Can the World Impose Logical Structure on Language?' in *Wittgenstein — Eine Neubewertung. Akten des 14. Internationalen Wittgenstein-Symposiums*, R. Haller and J. Brandl, eds. (Vienna: Hölder-Pichler-Tempsky 1890), 25-6.

16 Brian McGuinness, 'The So-Called Realism of the Tractatus,' in *Perspectives on the Philosophy of Wittgenstein*, I. Block, ed. (Cambridge, MA and London: MIT Press 1981), 65-6.

sight, this seems to suggest that the concept of object for Wittgenstein is a concept defined by logical concepts alone. However, he gives a negative answer to the question, 'is ''name'' so to speak a logical concept?' There he says that names signalize 'what is common to a single form and a single content' (NB, 52). A similar conceptual structure can be found in the *Tractatus*. According to TLP 3.201 and 3.203, objects are those entities, to which names ('simple signs'), in completely analyzed sentences, refer. However, in TLP 3.202 names are defined as the simple signs *used* in a sentence. This presupposes that the sentence sign itself is used, i.e. is a proposition (TLP 3.5-4). Propositions, for Wittgenstein, are pictures of possible states of affairs (TLP 2.201; 3-3.02), which means that they are pictures of possible concatenations of objects (TLP 2.01). In this way, the concepts of object and state of affairs enter into the definition of the concept of proposition. The definitions suggested by Anscombe, Ishiguro and McGuinness are affected by this conceptual structure. Although it is true that, following Anscombe, objects can be defined through the syntactical role of names, the concept of name is not a purely logical concept, but, in turn, presupposes the concept of object. Moreover, if the definitions proposed by Ishiguro and McGuinness are understood in a literal sense, they are clearly wrong. Not all subject terms of completely analyzed sentences stand for objects: subject terms in mathematical or logical sentences do not refer to mathematical or logical objects (TLP 4.441; 5.4; 6.02). For the same reason, not all mutually substitutable signs in completely analyzed sentences are names of objects: mutually substitutable signs in mathematics or logic do not have reference at all. In order to get a correct definition of the concept of object, the definitions suggested by Ishiguro and McGuinness have to be relativized to propositions, which excludes mathematical and logical sentences (see TLP 5.534; 6.1263). But in this case, the interdependence of logical and ontological concepts comes up again: the concepts of state of affairs and object enter into the definition of meaningful sentences.

The interdependence of logical and ontological concepts also becomes evident in Wittgenstein's remarks on variables. Variables, according to him, function as 'proto-pictures' [*Urbilder*] of simple objects. Since variables are propositional variables, they can be expressed independently of a sign such as 'x.' In this sense, there does not have to be a sign for a given proto-picture. As Wittgenstein explains in the *Notebooks 1914-1916*:

> That there are no signs of a particular proto-picture [*Urbild*] does not show that this proto-picture does not exist. The picturing by means of a sign-language does not take place in the way that a sign of a proto-picture stands proxy for an object of the same proto-picture. The sign together with the internal relation determines [*bestimmt*] the proto-picture; like the basic co-ordinates together with the ordinates determine the points of a figure. (NB, 46)

Here it becomes clear that Wittgenstein does not intend to give an account of concepts such a simple object in purely syntactical terms. Rather, he holds that the internal relations between linguistic expressions and reality are constitutive for variables. Variables are 'proto-pictures' that go proxy for objects only because they express structural similarities between propositions that stand in internal relations to reality. The connection between propositional variables and the concept of object has two interesting consequences for Wittgenstein's logic of picturing. On the one hand, thinking about the concept of simple object in terms of a theory of formal concepts explains why Wittgenstein is non-committal as to the qualitative nature of simple objects. Formal concepts, understood as expression of formal similarities of propositions, are capable of expressing only the formal side of the constituents of propositions and their referents. On the other hand, understood from the perspective of his remarks on variables in the *Notebooks 1914-1916* and the *Prototractatus*, it becomes clear the notion of a formal concept does not reduce to a purely syntactical account of concepts such as 'object'. Rather, if formal concepts are expressions of propositional variables, the concept of internal relations between propositions and reality is built into the notion of a formal concept. Indeed, Wittgenstein points out the impossibility of making a strict separation between the logical and the ontological: 'A formal concept is given immediately any object falling under it is given. It is not possible, therefore, to introduce as primitive ideas objects belonging to a formal concept and the formal concept itself' (TLP 4.12721).

V Conclusion

In a famous retrospective remark from 1931, Wittgenstein writes: 'I think there is some truth in my idea that I am really only reproductive in my thinking. I think I have never *invented* a line of thinking but that it was always provided for me by someone else & I have done no more than passionately take it up for my work of clarification. That is how Boltzmann[,] Hertz[,] Schopenhauer[,] Frege, Russell, Kraus, Loos[,] Weininger[,] Spengler, Sraffa have influenced me' (CV, 16-17). Several points deserve notice here. First, Wittgenstein originally wrote, 'This is how Frege, Russell, Kraus, Spengler, Sraffa have influenced me,' and added the names of Boltzmann, Hertz, Schopenhauer, Loos, and Weininger later. This should give at least a hint to the relative importance Wittgenstein himself assigned to the influence of each of these authors. Second, in this passage, Wittgenstein not only says that the authors he mentions influenced his work. He also adds, 'What I invent are new *comparisons*' (CV, 16). Moreover, a little later in the same manuscript he

writes: 'Anyway, when I was in Norway during the year 1913-14, I had some thoughts of my own, or so at least it seems to me now. I mean that I have the impression of having given birth to new lines of thinking at that time (But perhaps I am mistaken). Whereas now I seem just to apply old ones' (CV, 16-17). This suggests that Wittgenstein's early attitude towards Hertz's mechanics should not be seen in the perspective of a simple adaptation of foreign theories. Rather, it should be seen in the perspective of a comparison of ideas stemming from the pre-*Tractatus* period, which seemed new to Wittgenstein, with ideas that served as illustrations for what he had in mind. The comparison he draws between his view of the method of logic and the method of Hertz's mechanics should be taken seriously as a comparison between a genuine Wittgensteinian insight into the construction of propositions and Hertz's view of the construction of the propositions of physics. Moreover, seen from this perspective, it becomes clear why neither the *Principles of Mechanics* nor the *Tractatus* contains any (positive or negative) assumptions as to the qualitative side of the ultimate constituents of reality. For Hertz, as for Wittgenstein, the ultimate constituents of reality are characterized only according to their formal side because they are introduced as what guarantees the sameness of the logical multiplicity of both representation system and reality. According to Wittgenstein, all that can be said about the nature of simple objects is that they are the referents of names in propositions with a sense, and that their formal properties are mirrored by the formal properties of the possibilities of the combination of names into propositions. At the same time, since there is no purely syntactical characterization of a proposition with a sense, what a name is cannot be specified independently of its functions of denoting a simple object. In this sense, the concept of simple object is a formal concept — a concept that, once it is formed, cannot be understood independently of the concept of the object falling under it. The concept of a variable and the concept of a simple object, thus, are not independent of each other. Due to its connection with the role of variables in the generalized propositions of physics, the concept of material point forms a part of a net of interrelated concepts in which neither logical nor ontological concepts can be understood independently of each other.

Received: October 2005