

Critical Notice

DANIELLE MACBETH, *Frege's Logic*. Cambridge: Harvard University Press 2005. ISBN 0-674-01707-2. Pp. x + 206.

I A Focused Exposition

This is an engaging, controversial, and refreshingly well-written book about Frege's logic from the *Begriffsschrift* to the *Grundgesetze*. Danielle Macbeth explains Frege's complicated two-dimensional logical notation more patiently, accurately, and with a greater variety of examples than I have previously seen. She does so, moreover, not merely by relating Frege's innovations to the linear *Principia Mathematica* style logical symbolism that most logicians these days take for granted, but by delving into the deeper reasons why Frege choose to represent logical relations in this special pictorial way.

II Organization of Themes and Topics

The book is divided into five main chapters of four-five sections each, preceded by a Preface and Introduction, and followed by an Epilogue, notes, list of abbreviations and index. The chapters describe a historical progression in Frege's thinking about the expressive capabilities of the original *Begriffsschrift* logic, as they introduce 'The Starting Point,' and cover a succession of topics on 'Logical Generality,' 'A More Sophisticated Instrument,' 'The Work Brought to Maturity,' 'Course of Values and Basic Law V.'

The chapters track Macbeth's inquiry into specific problems in Frege's *Begriffsschrift* logic. She begins with a detailed examination of the specific concept of generality Frege sought to formalize. She proceeds to Frege's notion of function and higher-level generalizations (functions of func-

tions) first recognized in advanced mathematics. This background leads her to an account of Frege's logicism and the use of the *begriffsschriftlicher* symbolism with three orthographic styles of variables for three levels of generalizations required for the logical foundations of mathematics. Macbeth's discussion culminates finally in a discussion of Frege's later distinction between *Sinn* (sense) and *Bedeutung* (reference, meaning), in relation to the so-called Julius Caesar problem, and the disastrous inclusion of Basic Law V in the *Grundgesetze der Arithmetik*. This is the principle concerning the extensions of predicates and the comprehension of objects by *begriffsschriftliche* descriptions of property combinations, against which Russell's famous paradox of 1901 was directed.

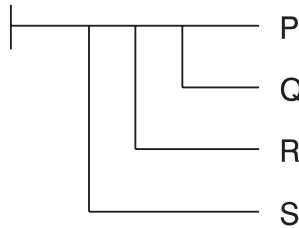
After preliminary discussion of what makes Frege's logic distinct from contemporary formalisms, I consider four interrelated subject areas that Macbeth emphasizes as central to her new interpretation of Frege's logic: (1) Frege's evolving conception of logical laws, which Macbeth distinguishes from generalized conditionals and characterizes as *inference licenses*. (2) The proper interpretation of Frege's *Begriffsschrift* and *begriffsschriftliche* notation in the *Grundgesetze*, including Frege's use of three orthographic styles of variables in generalizations. (3) The question of whether Frege's *Begriffsschrift* constitutes a quantificational or proto-quantificational logic sufficiently similar to linear *Principia*-style predicate-quantificational systems. (4) Frege's post-1891 distinction between *Sinn* and *Bedeutung*, presented in relation to his later understanding of the nature of a concept and the relation among concepts expressed in generalized conditionals.

III Uniqueness of Frege's Logic

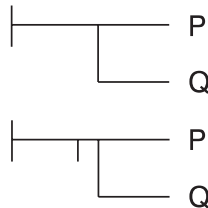
Macbeth encourages the reader to appreciate the uniqueness of Frege's system as a different kind of logic than the *Principia* logics it later inspired. Whereas it is too easy otherwise to regard Frege's concept-script as a crude idiosyncratic prototype of linear formulations in mathematical logic, Macbeth challenges the conventional wisdom that Frege's logic is a strange-appearing precursor to ordinary quantificational logic. She emphasizes two features that have not otherwise received adequate attention in critical commentary on Frege.

First, Macbeth explains Frege's use of the two-dimensional display of propositional logical connectives. She argues that Frege's notation is not merely an eccentric way of formulating primarily the material conditional and negation in forms that are logically equivalent to the more familiar propositional connectives including conjunction, disjunction and biconditional, but more significantly as a way of pictorially representing the *subordination of concepts* one to another. As emblematic of the

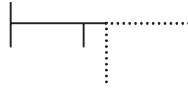
essential distinctions between Frege's and more contemporary logic that prevent the two from being regarded as mere notational variants, Macbeth maintains that in Frege's notation, unlike linear logics, there is no main connective (50-1). This may seem a startling claim, and Macbeth does not formulate the point as carefully as she might, but the idea seems to be this. In the two-dimensional *Begriffsschrift* notation, sentences involving three or more propositional symbols (or occurrences of the same propositional symbol) can be translated into alternative linear *Principia*-style expressions with different main connectives. Thus, Frege's *Begriffsschrift* sentence (F),



can be understood in standard propositional logic, among other ways, either as (a) $S \supset [R \supset [Q \supset P]]$ or (b) $[S \wedge R \wedge Q] \supset P$. Since in (a) but not in (b) the main connective is a conditional with S as its antecedent, Macbeth concludes that (F) unlike (a) or (b) has no main connective as such. Of course, the two readings are logically equivalent; otherwise Frege's logic would be inconsistent. The fact that there are two different *Principia*-style formulations of (F) is nevertheless taken by Macbeth as showing that a single sentence in Frege's two-dimensional notation displays multiple pathways of logical dependency in which no particular link stands out univocally as the expression's main connective. Macbeth's reasoning in contrast does not seem to apply to the simplest *Begriffsschrift* formulas such as



in which a main connective is discerned. Indeed, we can say, more generally, for any *Begriffsschrift Satz* in which the negation sign is in the leftmost place, of the form



that negation is in fact the main connective. The family of such counterexamples contradicts Macbeth's claim in its full generality, but does not detract from Macbeth's interesting observation that complex sentences like (F) do not have a main connective in contradistinction to every complex (non-atomic) linear *Principia*-style well-formed formula. As Macbeth writes in n. 5 to this chapter: 'We could, of course, dispense with the sign "&," using only signs for negation and the conditional. What is critical is the order of embedding of connectives, the fact that any of the connectives in " $S \supset (R \supset (Q \supset P))$ " can be made the main connective, given appropriate transformations' (188).

Second, Macbeth characterizes the multiple levels of quantificational domains Frege was concerned to represent in his early formulation of a functional calculus. These provide in effect a typed logic of first- and higher-order (second and third 'level') logics. Each is marked by a distinct choice of Latin, Greek, and gotische-Fraktur German (hereafter, 'Gothic') letters of the alphabet in the semicircular depression well that are inscribed along the content-stroke (*Inhaltstreich*) or horizontal lines of *Begriffsschrift* generalizations. These variables function logically like their counterparts in linear *Principia*-style quantificational logics, binding orthographically identical occurrences of variables attached to predicates within their respectively distinguished scopes. Why not then follow mainstream opinion by regarding Frege's logic as the first groundbreaking quantificational system?

IV Fregean Logical Laws as Inference Licenses

Macbeth advances a central interpretive thesis concerning the nature of logical laws in Frege's logic. Frege's *Begriffsschrift* and *Grundgesetze* on Macbeth's reading portray logical laws as generalized conditionals, but she insists that in another sense they are importantly different. She construes Fregean logical laws as *inference licenses* from which generalized conditionals follow. She begins with the recognition that inference rules in a manageable logic should not be multiplied beyond practicality, let alone necessity: 'In order to keep to the bare minimum the modes of inference employed,' she writes, 'all rules of inference — save for *modus ponens*, which is required if anything at all is to be inferred — are to be expressed in *Begriffsschrift* as formulae. This is the fundamental idea of

Frege's notation. It is motivated, we will see, by the project of logicism as Frege conceives it' (9).

This opening move is peculiar. Frege distinguishes between axioms such as contraposition expressed as propositions or principles and *modus ponens* as the one and only rule of inference. Thus, it is not clear what Macbeth means to refer to in speaking of 'all rules of inference — save for *modus ponens*.' Understanding all of Frege's axioms except for *modus ponens* as 'inference licenses' would seem dramatically to step away from Frege's deliberate efforts to have a single principle of inference combined with logical axioms. Nor is it evident what might be gained by characterizing Frege's logical axioms as inference licenses when they function perfectly well as truths of logic from which inferences can be drawn exclusively by *modus ponens*.

What is further obscure in Macbeth's reasoning, though arguably not in Frege's original theory, is why an exception should then be made for *modus ponens* as something other than an inference license. This is particularly hard to understand in light of the fact that any of Frege's axioms might be alternatively expressed as a rule of inference while *modus ponens* might be written out instead as a single proposition or axiom. One senses that the distinction in Frege is not hard and fast, but rather a matter of simplicity, elegance, convenience, or even tradition. Whitehead and Russell in the *Principia* were later to blur the difference entirely by collapsing them into the 'Primitive Propositions.' A logic tasked with the problem of deriving mathematical propositions should in some sense authorize the detachment of stand-alone conclusions. There is no obvious reason, however, why such conclusions could not themselves be conditional, reflecting at least some of the assumptions on which they rest in their antecedents. If Frege's model had been of that conventional sort instead, then a rule such as hypothetical syllogism might be considered an equally good candidate to stand on its own as the primary rule of inference in the system, which could certainly be expressed as a generalized conditional.

It is incumbent on Macbeth, given her interpretation of Frege's purpose, to explain why he does not make such choices. Even if we accept Macbeth's argument concerning Frege's uses of the *Begriffsschrift* in its most general terms, there is no obvious reason why *modus ponens* should be preferred on these grounds to other basic inference rules like *modus tollens*, or, say, *reductio ad absurdum*. Moreover, Macbeth's strategy leaves something interesting unexplained. If she is right that Frege treats logical laws as generalized conditional inference licenses, then why should there be exactly one exception from this formalization in the case of *modus ponens*? Why, indeed, except for aesthetic considerations that Macbeth does not explore or even mention for consideration, should

Frege have made an exception only for detachment by *modus ponens* as a logical law that is not formalized as an inference license?

Macbeth, moreover, does not offer any examples or quotations from Frege's writings to document or justify exceptional treatment for *modus ponens*. The only illustrations she provides of logical laws from Frege's texts are in every instance conditional. In the single citation she offers (18) in relation to this theme from Frege's *Collected Papers*, she quotes Frege as speaking only of normative 'laws of truth' in contrast with descriptive 'laws of nature', although the passage is introduced by Macbeth and commented upon as though it unquestionably applied directly to logical laws. Frege might reasonably include laws of logic in his classification of 'laws of truth', or in any case he might reasonably be construed as doing so, but that by itself does not imply that, with the surprising exception of *modus ponens*, logical laws are generalized conditionals that function as inference licenses. Macbeth argues:

Laws, Frege thinks, are fundamentally different from facts. They are not prescriptions, but prescriptions follow from them. Such laws are expressed in *Begriffsschrift* as generalized conditionals, that is, in sentences that on the standard reading of Frege's notation could equally well be written using the universal quantifier and horseshoe. This reading is called into question by three related and recurring themes in Frege's writings: that the conditional stroke is justified as the primitive sign for a logical notation in *Begriffsschrift* by its role in the formation of generalized conditionals and only generalized conditionals, with one significant exception, are correctly rendered in natural language using 'if ... then'; and that causal laws are expressed in *Begriffsschrift* as generalized conditionals. All three themes are sounded already in the early essay 'On the Aim of the "Conceptual Notation"' (19).

Here Macbeth mentions again the 'one significant exception,' presumably for *modus ponens*. The passage promises three reasons why Frege cannot regard logical laws merely as generalized conditionals of any and every type. A close reading of the long quotation from Frege's essay (CN, 95) to which Macbeth refers nevertheless says nothing whatsoever by way of making an exception for *modus ponens*. On reflection, furthermore, it sits rather badly with Frege's standards of rigor to consider all logical laws *but one* as belonging to a given category. Why the special treatment for just this one standard rule of inference, particularly when Frege does not describe it as such and arguably does not need any exceptions at all?

Macbeth's assertion that Frege understands logical laws as inference licenses belonging to a special category, rather than merely as generalized conditionals, is potentially one of the most exciting parts of her book. Unfortunately, she offers no shred of supporting evidence from Frege's writings to substantiate the claim that Frege himself ever thought of the laws of logic in just this way. The doctrine appears to be an

addition of Macbeth's own to Frege's philosophy. She advances the proposition as an interpretation, apparently as the best explanation of what Frege is about. She nevertheless does not make this part of her methodology explicit, nor does she consider alternatives for the sake of comparing her analysis with other accounts, assemble the kinds of data her thesis supposedly better accounts for than rival explanations, or engage in the kind of argumentation that most readers will have come to expect from this mode of defending an unusual and for that reason potentially contentious hypothesis. The examples in the selection she quotes from Frege's 'On the Aim of the "Conceptual Notation"' do not involve or require the detachment of any conclusions as a part of the logic itself, although its applications even within mathematics might be another matter. Preserving the antecedents of the conditionals by not detaching their consequents through *modus ponens* even in pursuing a logical reconstruction of arithmetic by means of axioms or inference principles seems more in keeping with Frege's idea for the *Begriffsschrift*, in that it permits each use of the logic explicitly to exhibit its inferential pedigree. The proposition that appears directly opposite the short vertical judgment-stroke or *Urteilstrich*, at the far right end of the uppermost content-stroke or *Inhaltstrich*, is already in a graphic sense the 'detached' conclusion of each conditional in Frege's two-dimensional notation. It is only tautologies that are unconditional in the system, whereas the truth of any and every other category of proposition is conditional on the truth of other propositions. The conclusions distinguished in this way can be assembled into extended chains, in which the end result of the process is a perspicuous exposition of the logical relations by which the propositions needed for the foundations of arithmetic and other branches of mathematics are vividly displayed.

Ironically, Macbeth, by reinterpreting Frege's axioms as inference licenses and making an unsupported exception for *modus ponens*, unconsciously slips back into the habits of interpreting Frege's logic that she condemns. For the use of *modus ponens* to detach the consequents of conditionals where the antecedents obtain is among the most conspicuous features of the first-order propositional and predicate-quantificational logics she wants to avoid confusing with what she takes to be Frege's characteristically different approach. On (58), for example, Macbeth invokes *modus ponens* again in connection with the need for real variables in drawing inferences from generalized conditionals as 'essential in logic as Russell understands it.' This is certainly true for Russell — but for Frege? The question is by no means trivial, but goes to the heart of Macbeth's account of the purpose of Frege's logic. Surely Macbeth is wrong also in this context when she argues that 'the truth of a thought is necessary and sufficient for correctly acknowledging its truth...' (33). Necessary, yes; but hardly sufficient, even if we presuppose the existence

of minds, which must additionally be disposed in a particular way for the acknowledgement of any true thought actually to occur. Frege, admittedly, is an objectivist of a roughly Platonic sort about the nature of meaning, reference and truth, but not, it would appear, about the mind's recognition or acknowledgement of the instantiation of any of these semantic properties.

On another topic related to his early logic, Frege's limitation of inference to whatever can be derived from true assumptions (3; 26-7) finally seems to provide the basis for if it does not actually constitute a kind of relevance logic. Together with a corresponding deduction theorem, the restriction neatly and with good philosophical rationale avoids the so-called paradoxes of material and strict implication. It would have been interesting for Macbeth to have explored this aspect of the *Begriffsschrift* as part of her critical exposition.

V Three Styles of Variables in Frege's Logic

While Frege might want to preserve a distinction between generalized conditionals and logical laws, in which logical laws are generalized conditionals but not all generalized conditionals are logical laws, and only logical laws are, in Macbeth's terminology, 'inference licenses,' there is no clearcut basis in anything Macbeth proposes by which Frege is syntactically equipped to distinguish logical laws as inference licenses from run of the mill generalized conditionals (see 30-2; 34-5). This problem leads to the next major topic of interest in Macbeth's treatment of Frege's logic.

Where if at all is the distinction between generalized conditionals and logical laws to be drawn in Frege? How might the distinction be established, even looking beyond Frege's writings, if we are not to allow as Macbeth seems to do that any and every generalized conditional can in principle serve as an inference license, on pain of eroding the distinction Macbeth wants to sustain between mere generalized conditionals and logical laws? If we are being charitable to Russell in his struggles to come to terms with Frege's early logic, we can sympathize with the difficulty of demarcating the difference between logical laws and generalized conditionals precisely for this reason. One might then further imagine that Frege in the *Begriffsschrift* sought to represent their distinct categories by the difference in styles of variable, particularly between the Gothic and italicized Latin variables respectively in the specific case of generalized conditionals as logical laws and 'judgments' more generally. This clearly does not suffice, and it would be astonishing if such magic could be accomplished merely by introducing a difference in variable

style. What, then, does Frege hope to achieve by distinguishing between three types of variables in the formalization of logical generalizations?

Italicized Latin variables, for Frege, as Macbeth rightly observes (62-5), do not designate at all, but serve only as convenient abbreviations, as when we say in algebra, for example, 'Let $x = 1$,' without bothering to preface the remark with 'For any x ' or 'For some x .' Such variables are never bound by like variables in the scope-delimiting concavities (the semicircular dips or wells in the content-strokes) of *begriffsschriftliche* generalizations. Frege does not seem to regard them as serving any function other than the heuristic one of bridging the gap between the *Begriffsschrift* and conventional mathematical languages. Greek variables in turn appear primarily as place-holders for substituends in function expressions and occasionally as variables, typically unbound as in standard first-order predicate-quantificational logic, in the names of specific 'courses of values.' Type distinctions, such as they are in Frege, are indicated only by the difference between lower- and upper-case Gothic variables, the former for objects and the latter for concepts interpreted as unsaturated functions in need of completion by the insertion of arguments.

Where Frege's uses of the two primary (Gothic and italicized Latin) styles of variables are concerned, Macbeth recognizes that Frege in the *Begriffsschrift* seems to limit not only logical laws but properly expressed meaningful generalizations of any kind as involving bound Gothic variables. Free variables, regardless of script, but certainly in the case of italicized Latin letters outside of Frege's special dispensation, as one of my sainted logic teachers used to say, are an invention of the devil. Macbeth in any case is not the first to interpret Frege's multiple alphabets of variables in this way. It has long been recognized that Frege offers at least a limitedly typed logic in which quantification (or generalization) is permitted over lower-order objects and higher-order functions for the expression of certain mathematical derivations. The latter in particular are formalized in two ways in Frege's later system, as capitalized Gothic letters, and as the lower-case Gothic equivalent of the letter 'f,' specially so designated as a mnemonic for 'function,' where both are typically used as bound variables appearing in the concavity of a generalization's content-stroke. Frege could hardly be more explicit about the matter, at least in the *Grundgesetze*, where he applies a distinction in his syntax that he seems to have fully appreciated only some time after the publication of *Begriffsschrift*, in which the symbolism for different styles of variables is already in evidence. Macbeth's account plausibly describes an emerging awareness on Frege's part as to why he introduced a distinction that he was only to make use of in a somewhat unexpected way later in his career. What it leaves unexplained, in an area where perhaps no authoritative or fully satisfying explanation is forthcoming, is how he managed

to have the raw luck or unconscious insight to include a distinction at the early stages in designing his logic for which he discovered a need only many years down the road. Perhaps Frege only dimly perceived the importance of a distinction in types of variables, which he developed and utilized when its real advantages became apparent during the course of his thinking culminating in the *Grundgesetze*. Stranger things have happened in the history of logic as in other fields of human endeavor; still, I find the need to resort to this kind of explanation, despite the documentation Macbeth assembles from Frege's correspondence, to be particularly unsettling in Frege's case.

VI Frege's Logic as Proto-Quantificational

Macbeth not only resists but remonstrates against speaking of Frege's logic as 'quantificational.' In part, it seems, she does this to mark a sharp break from a history of interpretation that has been unable to read Frege's advances in logic in their own light instead of through the lens of what have emerged as more fashionable *Principia* logics. The tradition of misunderstanding Frege's logic according to Macbeth goes back to Russell, who was first introduced to Peano's quantificational logic at the 1900 World Congress of Philosophy in Paris.

Peano's logic, as Russell reports, was a revelation. It changed his philosophical outlook so profoundly as to permanently set many of his formal expectations in stone. Later, as a result, Russell had to struggle unsuccessfully to translate Frege's significantly different formalizations through the filter of Peano's linear logical symbolism that was ultimately to inspire the *Principia* and its heirs. Macbeth quotes Russell as remarking frankly in a 1910 letter to Philip Jourdain: 'I could not understand Frege's use of Greek, German, and Latin letters, and I put him away for nearly two years, by which time I had discovered for myself most of what he had to say, and was therefore able to understand him' (Macbeth 6).

This is an historically important admission, as Macbeth interprets the remark, because it identifies the point at which Russell has already gone too far in another direction to interpret Frege in Frege's own terms. Although Russell acknowledges that at first he did not understand Frege, he claims later to have been able to have done so. If Macbeth is right, however, then Russell eventually comes to 'understand' Frege only in the sense of projecting onto Frege's formalism a very different medley of purposes, priorities, and expectations about what such a logic should involve. From Macbeth's perspective, Russell does not later understand Frege, but reads into Frege's logic many of his own ideas derived in part from Peano that do not always agree with Frege's original intentions. If Macbeth is right, then the confusion of Frege's concept-

script with *Principia*-style logic after Russell is if anything even more prevalent in the later secondary literature. Frege himself labored to distinguish his logic from Peano's in his 1897 essay (Macbeth mistakenly gives its publication date as 1896), 'Über die Begriffsschrift des Herrn Peano und meine eigene' ('On Mr. Peano's Conceptual Notation and My Own'). Macbeth wants to set the record straight by avoiding assimilations of Frege's logic to received systems that have taken their inspiration directly from Russell and indirectly through Russell from Peano. Thus, Macbeth concludes her Introduction to the book with these cautionary words:

What Russell discovered, of course, was, first, that Frege's logic could — though with difficulty — be read as a notation of Russell's own logic, and also that the contradiction [Russell's paradox] was derivable in Frege's system as well. We have been reading Frege's notation as a notation of quantificational logic ever since. (7)

We find statements in many sources of what Macbeth regards as mistaken or misleading accounts of Frege's logic as a quantificational system. An example that Macbeth does not cite but that fits her argument exactly appears in Erich H. Reck and Steve Awodey's recent translation of *Frege's Lectures on Logic: Carnap's Student Notes, 1910-1914*. There in the Introduction the editors write: 'The system of logic presented in the lectures is a hybrid fragment of the systems presented in Frege's published work ... it is essentially what we would now call a system of higher-order, predicate logic, similar in many respects to the system of Whitehead and Russell's *Principia Mathematica* (1910/1913), but with "simple" rather than "ramified" types'. (Chicago and Lasalle: Open Court, 2004. Publications of the Archive of Scientific Philosophy, Hillman Library, University of Pittsburgh, 29)

Macbeth's study challenges this mainstream interpretation of Frege's logic as proto-quantificational. Whether or not Frege did in fact devise a quantificational logic, Macbeth believes, is a long overdue question whose answer in the affirmative should not be taken for granted. She indicates at the beginning and end of the book that she is primarily interested in having her readers consider seriously and for the first time whether Frege's logic is a quantificational system as we have come to think of it. In between these guarded appeals for others to join in asking the otherwise neglected question of the book, Macbeth addresses the issue by holding unequivocally that Frege did not devise a quantificational logic in the sense the term has come to have.

Logicians, historians and philosophers of mathematics have been offering this purportedly misleading characterization of Frege's logic for almost a century. What, exactly, does Macbeth find wrong with such pronouncements? The main objection she offers is that a quantificational

logic in the accepted sense of the term does not distinguish as Frege does between laws and accidental generalizations, while Frege's three styles of variables function differently than bound variables in first-order predicate-quantificational logic. Russell did not grasp these subtleties, if Macbeth's reading of the history of Frege's reception is correct, and Russell set the tone for later misinterpretations that she proposes to set right. It was Russell, after all, who introduced Frege to the Anglo-American philosophical community. Russell popularized an otherwise obscure, virtually unread Frege, even in the German philosophical community, and made him a household word around later analytic campfires as the pioneer of a logic that would achieve its apotheosis in the *Principia*. Generations of Russell's admirers, who otherwise are unlikely to have heard of Gottlob Frege, read Frege as Russell afterward did, honoring him as the great grandfather of the functional calculus or first-order predicate logic, together with a handful of valuable distinctions in philosophy of language and philosophical semantics that Macbeth believes were almost never fully appreciated or properly integrated into the historical fabric of Frege's evolving thought.

Frege, whose *Begriffsschrift* and *Grundlagen der Arithmetik* were ignored, or, in the latter case, negatively and damagingly reviewed during his lifetime by respected contemporary authors such as Georg Cantor, had to pay out of his own pocket to have the two volumes of the *Grundgesetze der Arithmetik* privately published. Frege did not have the kind of following or established school of interpretation to oppose what someone with Russell's reputation and influence in the professional community had to say about the meaning of what was after all supposed to be an ideally perspicuous logical notation. Moreover, Frege's logic was so revolutionary and comparatively difficult to master that Russell's experience of 'understanding' it only after he believed he had rediscovered its original principles by following another path has been frequently reiterated by students of Frege viewing his lifework from a distinctively Russellian perspective. Macbeth adds more pointedly:

The full polyadic predicate calculus was Russell's contribution, an extension of Peano's logic following [C.S.] Peirce's work on the logic of relations. Only later, after he had developed quantificational logic, did Russell read that logic back into Frege's astonishingly ill conceived notation. Russell simply assumed that Frege had independently developed essentially the logic that Russell had developed, and in the heady early days of the discovery of quantificational logic that assumption would have been quite natural. We have no such excuse. Is the logic that Frege first developed in 1879 and, by his own account, brought to maturity with the introduction of the distinction of *Sinn* and *Bedeutung* in the early 1890s a quantificational logic? The question has not been asked. Yet it must be asked. For the simple fact of the matter is, we do not know. (7)

All this, as far as it goes, is sound exegetical strategy on Macbeth's part. It is reasonable to raise these concerns, particularly in light of Frege's own efforts to distance his logical symbolism from Peano's. Macbeth may nevertheless go too far in denying or even merely hesitating to acknowledge that Frege's logic is quantificational. I find it more natural to describe this part of Frege's project as a special kind of type-restricted quantificational logic in a category of its own.

Frege's logic is often understood as quantifying over ordinary objects (roughly, Aristotelian primary substances and Platonic abstract mathematical entities), and by extension, eventually, over functions, including concepts taken as higher-order objects. That is unproblematically *a* (kind of) quantificational logic. Macbeth is nevertheless right, given the differences between standard predicate-quantificational logic and Frege's built-in restrictions on quantifications that can belong to the *Begriffsschrift*, to question whether it is *the* quantificational logic that follows the line of succession from Peano and Peirce to Russell and beyond. More importantly, Macbeth is on firm ground to raise the issue of whether and if so in what exact respects Frege's logic has affinities with and differences from what has become the standard *Principia*-style quantificational logic that most of us have been taught. Macbeth interprets the concavity in Frege's notation, not as representing a quantification over objects, but rather as designating a second-level property that might be glossed as being universally instantiated, and under which a first-level concept is subsumed. Again, what is wanted in Macbeth's reading is the necessary documentation to support an interpretation that runs against the grain of more straightforward explanations. How, in particular, are we to understand the concavity containing α as attributing a second-level property to a bit of syntax such as $F\alpha$, when $F\alpha$ by itself according to Frege is rightly not considered in itself to be a fully formed sentence or *Gedanke*? The problem becomes even more acute when the concavities are stacked up in different parts of a *Begriffsschrift* formula with what would otherwise be described as overlapping scopes.

We must nevertheless beware of our own presuppositions about logic that may color our reading of Frege, even if we are determined not to repeat Russell's particular kinds of hindsight misinterpretations of the predecessors to whom he explicitly acknowledges his intellectual debts. Macbeth offers a similar sounding concession, when she writes earlier in her Introduction:

Certainly Frege's notation can be read as a notation of quantificational logic, and any one of his "mistakes" taken alone might be excusable, albeit oddly inexplicable. But Frege makes too many mistakes on the quantificational reading, mistakes that seem to be completely arbitrary and often manifest. Is it really credible that Frege would have made such mistakes, and so many of them? If we think this, perhaps it is only because we can see no alternative. (5-6)

Macbeth makes a related but different point. She does not grant that Frege develops a (special kind of) quantificational logic, but that his logic *can be read* (presumably misread) as a quantificational formalism. If we follow the latter course, then Macbeth believes the interpretation will be fraught with so many 'mistakes' from the standpoint of modern quantificational logic that the principle of charity ought to induce us to withdraw the 'quantificational' reading altogether. Later in the book Macbeth says this:

Frege does often describe his notation for generality in ways suggestive of a Russellian reading, at least to anyone already familiar with that logic, but he also often describes that notation in ways that are incompatible with that reading. It is the latter passages we focus on here, and we do so because, *all* texts considered, it is the latter passages rather than the former that seem best to reflect Frege's most considered views. (63)

Macbeth then recalls Frege's understanding of Latin italic letters as variables in *Begriffsschrift* generalizations, pointing out that: 'They do not designate indeterminately; they do not designate at all. Their role is to confer generality of content' (63). This seems right, but from this Macbeth draws the surprising further conclusion that: 'Latin italic letters of *Begriffsschrift* seem not to be functioning as variables' (64). Let us grant that if Latin italics do not function as variables then at least those *Begriffsschrift* generalizations that contain Latin italics are not part of a quantificational logic modeled on *Principia* for which bound variables are essential. Even so it remains unclear that Macbeth's more general conclusion follows. In the first place, Frege seems to regard Latin italics in *Begriffsschrift* generalizations as a convenience rather than an essential element of the notation. Second, Macbeth seems to misunderstand Frege's remarks about a Latin italic's lack of sense or meaning in his notation, serving as it does only in 'conferring generality of content' to the expression of a thought (62). This, obviously, is a role that *Principia*-style variables can also fill.

More importantly, Macbeth thereafter seems to draw an obviously invalid inference. She writes, commenting on a passage from Frege's *Posthumous Writings* (190) to which she compares Frege's *Collected Papers on Mathematics, Logic, and Philosophy* (308-11) and *Philosophical and Mathematical Correspondence* (21):

If "*a*" were functioning as a variable, it would have a function in relation to the part [of a sentence expressing a thought]; indeed, that is the essential role of a variable, to confer content on the part in a way that enables the logic of truth-functions to be applied to a sentence containing it. Yet Frege denies that his Latin italic letters function in this way. On his view, a conditional written using Latin italic letters 'expresses a single thought which cannot be divided into component thoughts' (*CP* 309; see also *CP* 171). Because they have no function at all in relation to the part,

because they serve to confer generality only over a (conditional) sentence as a whole, Latin italic letters of *Begriffsschrift* seem not to be functioning as variables. (63-4)

The trouble is that Frege, given his philosophy of logic and language, could and probably should say precisely the same thing about the variables encountered in a *Principia*-style generalization or quantification. They also have neither sense nor meaning, and in such languages they also contribute only to the conferring of content on a sentence as a whole. Russell or another writer on philosophical logic might disagree, but that does not change the fact that the basis for regarding the *Begriffsschrift* as non-*proto*-quantificational on which Macbeth relies is compromised if Frege himself would not have recognized a distinction in how variables work in any formal system of logical generalizations.

Macbeth, searching for a concrete difference between Frege's logic and a *Principia*-style quantification, also seems to go too far when she argues that if Frege's generalizations 'cannot be divided into component thoughts' that therefore his Latin italics 'have no function at all in relation to the part'. How can this be true if a generalization can be divided into parts that are not themselves thoughts and if such parts in the meantime serve to confer content on the sentence as a whole? Macbeth seems to assume that the only relevant parts of a generalization in which Latin italics could function would need to be truth-functional. She is right that Frege's generalizations do not meet this test, but the same is true of *Principia*-style quantifications. Frege's assertion in the passage she quotes, in writing as he does, '*a* is greater than 2' we longer have a part expressing a thought,' is as true for *Principia*-style logics as it is for the *Begriffsschrift*, when, as Macbeth rightly declares, '*a* is greater than 2,' taken out of the context [reviewer's emphasis] of a conditional in which it occurs as the antecedent or consequent, is senseless' (63).

Exactly so. The same, unfortunately for Macbeth, is also true of any such fragment of a conditional also in a *Principia*-style logic, if we strip off the quantifier and binding variable in a formula such as $\forall x[Fx \rightarrow Gx]$, and consider only '*Fx*' or '*Gx*' in isolation or taken out of context in the same way that Frege considers for the sentence's *Begriffsschrift* counterpart. Since the variable '*x*' here, in and of itself, as Frege likes to say, lacks sense or meaning, it does not contribute truth-functional content to the expression in which it occurs, but its several occurrences are only involved in conferring content on the expression as a whole. This is a worthwhile observation about the contribution of syntax components to the generalizations expressible in a logic, but it applies equally to *Principia*-style logics and to the *Begriffsschrift*. As far as I can see, the argument does not provide a basis for distinguishing *Begriffsschrift* from *Principia*-style variables, and consequently provides no basis for distinguishing *Begriffsschrift* generalizations from *Principia*-style quantifications, or

therefore for denying that Frege's *Begriffsschrift* is a quantificational or proto-quantificational logic.

Macbeth's conclusion that Frege's logic is not quantificational as a result seems extreme. What I would prefer to say is that at most these difficulties arise only on one kind of quantificational reading, and that there need be nothing objectionable about discerning a quantificational logic in Frege's *Begriffsschrift*, provided we do not foist onto his formalism a kind of quantificational logic to which he does not ascribe. Nor do I think such considerations nitpicking in a context in which the problem at issue is to understand the exact nature of quantificational logic associated with Frege's project. The unique character of Frege's approach to logic is lost sight of if we conclude that it is not a quantificational logic on the grounds that it is not the kind of quantificational logic we are accustomed to seeing and comfortable working with. Macbeth is right to challenge efforts beginning with Russell to force Frege's logic into the mold of a proto-*Principia* quantificational logic. Her purpose is commendable in trying to understand Frege's system for what it is in its own right and on its own terms, bracketing as fully as possible the distorting presuppositions we may carry about as philosophical baggage in looking back at Frege's accomplishments. It is less clear whether in the process we need to exclude Frege's logic as non-quantificational, however appropriate it might be to recognize its uniquely heterodox character viewed from a contemporary perspective. We risk distorting Frege's work just as badly if we distance it too far from quantificational systems than if we lump them all together and fail to appreciate what is distinctive about Frege's generalizations.

VII Impact of the *Sinn-Bedeutung* Distinction

After 1891, with the publication of '*Über Sinn und Bedeutung*,' Frege was able to avail himself of another important distinction, one for which his philosophy of language is probably best known independently of his contributions to symbolic logic. In his famous letter to Edmund Husserl of 24 May 1891, Frege lays out the schema whereby he proposes to distinguish the sense and reference of proper names, concept words and propositions in approximately parallel fashion (reproduced by Macbeth, 117).

Macbeth identifies the significance of Frege's distinction between sense and reference in his theory of *begriffsschriftliche* generalized conditionals as expressing relations among concepts (139; 144-5). The distinction further permits Frege in his later period to overcome the limits of the early concept-object distinction whereby concepts equated with

functions are unsaturated, and hence not proper objects for higher-order predications. Macbeth maintains:

Concept words, on Frege's mature view, do not only express senses; they also designate concepts which, although unsaturated, are fully objective entities that can stand in logical relations one to another and can serve as arguments for higher-level functions. The laws of the special sciences, and the laws of logic as well, can then be taken to be fully objective, substantive truths despite the fact that they involve no reference to any objects, and in the case of the laws of logic no reference even to any first-level concepts. (140)

This reflection was essential for Frege's logicist program. Here he recognizes the need to generalize over functions of functions as third-level predications and quantifications in representing the logical foundations of arithmetic. Macbeth articulates three separate elements in coming to know a concept's *Sinn*, with particular application to the sense of an expression in the *begriffsschriftliche* logical formulations of mathematical principles:

First, one needs to know how to form and to recognize individual numbers such as '37' and '64722' and also more complex arithmetical signs such as '37 + 642' and '37⁴⁷'. One also needs to know how to apply such signs, in particular that zero is assigned to concepts under which nothing falls, and so on. Finally, one needs to know the algorithms of arithmetic, how to perform calculations according to the rules of arithmetic in order to prove identities.... Grasp of the thought expressed by a sentence in the formula language of pure thought, analogously, requires, on our view, first, the capacity to form and to recognize well-formed sentences in the language, second, the capacity correctly to judge of sentences in the language that they, or their negations, express true thoughts, and third, the capacity correctly to draw inferences in the language. (132, 153)

Irregularities in Frege's original understanding and applications of his *Begriffsschrift* may have necessitated or at least suggested to Frege the usefulness of the *Sinn-Bedeutung* distinction. Macbeth gestures toward this possibility in several places, but does not systematically develop what for the time being must remain an untold part of the story of Frege's philosophical self-realization. It would be useful in the history of logic for someone to consider a plausible hypothesis about exactly how Frege might have been driven toward the *Sinn-Bedeutung* distinction as a result of the limitations of the *Begriffsschrift* syntax for its intended applications in formalizing the principles of elementary arithmetic. Beyond a few hints, Macbeth unfortunately does not advance our understanding of this aspect of Frege's philosophy.

There are many questions in logic that may have impelled Frege toward his famous distinction. How is the identity sign supposed to be interpreted? What logical meaning does it have? What makes an identity expression true, and what makes it false? What are the object (or concept)

relata of identity relations? What do we need to quantify over in order to represent within the *Begriffsschrift* notation the exact properties of arithmetical concepts (functions) and objects? How can we symbolize such quantifications within the *Begriffsschrift*, and what do we commit ourselves to ontically and in other ways philosophically by choosing from a menu of different types of free or bound variables of different orthographic styles in order to represent this or that type of generalization? Why are there just three levels of generalization in Frege's *Grundgesetze*? Why just objects, concepts (functions), and functions of functions? How can quantification over concepts and functions occur at different (type) levels if concepts on Frege's view are themselves functions?

VIII Macbeth on Frege's Progress

We thus broach another central theme of Macbeth's interpretation. Among the valuable features of her account, Macbeth emphasizes the transitions in thinking Frege underwent during the course of his work in logic, particularly in his conception of the nature of logical laws and the distinction especially between Latin and Gothic variables. Macbeth clarifies their relation:

Frege came fully to comprehend the logical role of the concavity notation, and thereby the logical distinction between sentences expressed using his two notations for generality, only decades after the notation itself was developed. A *Begriffsschrift* genuine hypothetical expressed using Latin italic letters has the form of a subordination of concepts; expressed using the concavity notation, the sentence instead has the logical form of a subsumption of first-level concepts under a second-level concept. (90)

Frege not only arrived at a new application of his own distinction between variable lettering styles as his thinking progressed from the time of the *Begriffsschrift* through the *Grundlagen* to the *Grundgesetze*, but he underwent a three-stage development in his efforts to understand the nature of logical laws as contrasted with the laws of nature or science. Macbeth summarizes these phases:

At first, in *Begriffsschrift*, Frege seems to think of the laws of logic as merely more general than the laws of the special sciences, indeed, as maximally general but not in any relevant sense different from other laws. In the long Boole essay ['Boole's Logical Calculus and the Concept-Script'] written shortly after *Begriffsschrift*, and also in *Grundlagen*, Frege adopts the notion of form as it contrast with that of content to characterize the peculiarity of the laws of logic. The laws of logic, he now suggests, are fundamentally different from the laws of the special sciences because they are formal in a way those other laws are not. The third stage is reached with the idea, explicitly formulated only in 1906 in 'Foundations of Geometry II,' that the laws of

logic are at once contentful, just as the laws of the special sciences are, and qualitatively different from those other laws. Frege's final word on the matter thus seems to combine elements of both his earlier views. (96)

Frege's Basic Law V commits his logicism to the existence of a specific set for every formulable characterization of properties permitted by the *begriffsschriftliche* notation. Russell exploits the (excessive) strength of Frege's *Grundgesetze* Law V by describing a set of all sets that are not members of themselves, which is logically implied to be a member of itself if and only if it is not a member of itself. Later, Frege thought that the embarrassment could be avoided altogether by jettisoning Law V. He apparently had reservations about the principle all along and at one point evidently thought it was superfluous to the grounding of arithmetic by the logical principles formalized in the *Grundgesetze*. Critics of Frege's system have since seen its downfall as due to his inclusion of the extensionality principle, which they have argued is in fact indispensable to several of his important results and to the application of the *Sinn-Bedeutung* distinction to explain the meaning of all *begriffsschriftliche* sentences.

Macbeth further argues that Frege's comprehension-challenged *Grundgesetze* Basic Law V is needed at most only for Frege's logicism, not for his logic (176-7). She seems to prefer a logic that preserves what she perceives as the advantages of Frege's *Begriffsschrift* with the full range of alternative variable styles as Frege interprets them under the *Sinn-Bedeutung* distinction, while leaving extensionalist comprehension of the logic's domain aside.

This at first glance is admittedly an attractive way of seeing Frege's project in logical and philosophical context. The downside is that Frege shorn of logicism is like Aristotle without substance theory or Plato without the Forms. Frege is all about logicism — no logicism, no Frege. Indeed, Frege develops the *Begriffsschrift* logic entirely for the sake of his logicism. Frege would certainly never have been satisfied in his ambitions despite the loss of Law V, if he thought he needed it for the propositions of arithmetic and if he agreed that Law V was incompatible with his logic. Faced with this dilemma, he might reasonably have considered reexamining his logic rather than abandoning the aims of logicism. Any individual thinker has only so much time and so much energy to expend in such concentrated ways over the course of a life. There are many possibilities that Frege (and Macbeth) do not explore, and their conceivability — even if they would ultimately have been rejected by Frege or should independently be rejected by our own lights can reveal interesting unexamined facets of Frege's work as he understood it, to the extent that he was able to develop it and in the state he left it historically.

Why not, for example, qualify Basic Law V in such a way that Russell's paradox is forestalled? Did Frege think that any effort of the sort would be objectionably *ad hoc*? Was he convinced that any extensionality principle would have to take precisely the form of Law V as he originally stated it, so that to systematize extensionality at all would necessarily require including Law V in just its original form? Many of these questions have been taken up in the recent literature on Frege's logic, notably in John P. Burgess's 2005 study, *Fixing Frege* (Princeton University Press). Frege at one point early in his career seems to have thought that extensions and the extensionality principle were inessential, but that it was a matter of considerable convenience to include an extensional comprehension principle in explicating the logic of all but the most elementary propositions of arithmetic. Macbeth is very much in keeping with the later philosophical world's majority opinion that Frege's Law V was after all strictly needed at least for his logicism, and that Frege was mistaken in this judgment. The preponderance of Frege scholars currently hold that an extensionality principle for the meaning of concept terms is necessary if the *Sinn-Bedeutung* distinction is to apply to *begriffsschriftliche* formalizations of whatever *true* propositions can be expressed in the logic. This is particularly the case in arithmetic, but the general rule holds regardless more generally of whether or not such *Gedanken* happen to be expressed as sentences involving mathematical concepts. Many ontologists believe that extensions can be dispensed with paraphrastically in favor of intensions (property specifications) alone, and Frege's thesis of the dependence of *Bedeutung* on *Sinn*, effectively of extension on intension, might make him philosophically hospitable toward such a reduction, thus avoiding the need for Law V.

It might be too blunt to replace *Grundgesetze* Law V with a principle that establishes mere one-one correlations for like-numbered sets. Frege himself notes in the *Grundlagen* that such a principle by itself does not enable us to rule out as nonsense the proposition that Julius Caesar has all the same properties as the number 2. Macbeth (161-4) adds importantly that even if we could thereafter distinguish between Caesar and 2 on the basis of some proposal that does away with Law V, we would already need to be in possession of the concept of number in order to effect or even to make the proposed substitution intelligible. Macbeth astutely observes that Frege on pain of vicious circularity cannot take such a step within the context of trying to define the concept of number. This, however, only goes to show that not any and every replacement for the extensional comprehension principle in *Grundgesetze* Law V is sure to succeed. Without establishing that this is the only possible modification of or substitution for Law V, there is no further conclusion to be drawn about the possibility of rescuing another still recognizable form of Frege's logic and logicism.

Frege has no comparable difficulty with the senses and references of object terms. Thus, he has the alternative as well of relating only those objects that satisfy the otherwise unsaturated argument places of concept and function terms in such a way as to produce a possibly true complete thought. A modification of this kind preserves all of the logically, semantically and set theoretically unproblematic domain of a still recognizable *Begriffsschrift* logic and its viable logicist applications, while excluding problematic constructions like the Russell paradox that logically cannot possibly result in a true complete thought. Meanwhile, Frege could also choose to relax the distinction between objects and concepts or functions as unsaturated by allowing there to be saturated as well as unsaturated objects. Frege, as we have seen, is already prepared to quantify over higher-level concepts and functions, as well as functions of functions at second and third levels respectively, marked as distinct types with syntactically distinct categories of variables. Equally, then, Frege need not balk at the idea of treating concepts and functions as objects at least in the logical and grammatical sense when we attribute properties to them. Under such a modification of his original distinctions, Frege could then assert that such and such is the concept of identity and it is reflexive, or that this is the function of addition and it is commutative. With all of these and more options still available to him, we must wonder why Frege allows himself to be so defeated and then so confused by Russell's paradox. If we empathize with Frege's aspirations, and if we find high intellectual drama in his reaction to the letter by which Russell torpedoed the *Grundgesetze*, we may find ourselves asking which is worse — acknowledging the defeat of one's lifework in logic and philosophy, or being confused about whether and how it can still be salvaged?

IX Macbeth's More Fregean Frege

It is in unexpected ways a remarkably prescient Frege that we encounter in Macbeth's Frege. Frege, if Macbeth is right, is not the originator of modern quantificational logic as we have come to recognize it today. He nevertheless devises a kind of quantificational logic that unambiguously distinguishes quantifier scope at three type levels and permits generalizations involving objects, concepts and functions, and concepts or functions of functions. Whether Russell ever rightly understood the details of Frege's *Begriffsschrift*, there is no denying that Frege's logic in both its propositional and predicate-quantificational parts heavily influenced Whitehead and Russell's *Principia Mathematica*. Russell believed himself to have developed the functional calculus as an improved linear counterpart of the *Begriffsschrift* and *Grundgesetze*. Macbeth's study is impor-

tant because it moves beyond these platitudes to consider the differences between Frege and Russell, and to ask whether in seeing Frege always in retrospect through Russell's linear logic we are misperceiving something very different than Frege intended.

There is no doubt, as Macbeth emphasizes (49-50), that Frege's two-dimensional notation displays logical relations in ways and with theoretical advantages that are not available to *Principia*-style first-order logics. In this sense, Macbeth is right that Frege's *Begriffsschrift* is not simply an equivalent or eccentric precursor to what has since become the standard linear notation for first- and second-order predicate-quantificational logic. It is rather a distinctive logic belonging to a category of its own, designed for a particular analytic purpose related to the foundations of arithmetic in Frege's logicism. On the other hand, it is difficult in considering Macbeth's reasons for denying Frege as the originator of quantificational logic not to see the differences she emphasizes as minor and relatively unimportant. True, with qualifications, most of the differences to which she points are there; yet she does not say enough for a skeptical reader not to downplay these differences as unimportant in comparison with the remarkable similarities and their historical influence on such logicians as Whitehead and Russell, Wittgenstein, Quine, and the later tradition of first-order predicate-quantificational symbolic logic. Macbeth writes, in the Epilogue near the end of the book:

Having given up the uncritical assumption that Frege's logic is a quantificational logic, we were also able to render intelligible all the major developments in Frege's views, both those culminating in the distinction between *Sinn* and *Bedeutung* that constitute, on our reading, an extraordinary intellectual achievement, and Frege's misbegotten Law V.... Taken together, Frege's two-dimensional notation, with its Greek, Latin, and German letters, and his mature understanding of the logical functioning of that language in terms of the logical notions of *Sinn* and *Bedeutung* afford us the first glimpse of a world hitherto unimaginable.... Should we start with quantificational logic and aim to squeeze Frege's work into that mold — much as, on Frege's view, Peano aimed 'to squeeze each formula onto one line' (CP 236)? Or should we start instead with Frege's writings, letting quantificational logic fall where it may? The former path is well trodden. The latter might teach us something we do not already know. (179-80)

If these are our choices, then on purely interpretational grounds, I could not be more firmly in agreement with Macbeth that it is the second of the two paths she describes that is most likely to reveal what is distinctive about Frege's logic. The details of a correct account, however they are to be finally decided, should not detract from the fact that it is in Macbeth's innovative study that we are first challenged to try to understand Frege's lifework entirely on its own terms. That Frege did not develop a generically quantificational logic, on the other hand, as Macbeth maintains in her most original central thesis, is sure to strike

many readers as it does the present reviewer as an exaggeration that stands in need of significantly greater clarification and more convincing examination of primary sources.

DALE JACQUETTE
The Pennsylvania State University
University Park, PA 16802
USA

Netherlands Institute for Advanced Study (NIAS)
2242 PR Wassenaar
THE NETHERLANDS

