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The Role of Magnitude in Kant's Critical Philosophy

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Introduction

In the *Critique of Pure Reason*, Kant argues for two principles that concern magnitudes. The first is the principle that 'All intuitions are extensive magnitudes,' which appears in the Axioms of Intuition (B202); the second is the principle that 'In all appearances the real, which is an object of sensation, has an intensive magnitude, that is, a degree,' which appears in the Anticipations of Perception (B207).[1] A circle drawn in geometry and the space occupied by an object such as a book are paradigm examples of extensive magnitudes, while the intensity of a light is a paradigm example of an intensive magnitude. These principles justify and explain the possibility of applying mathematics to objects of

1 I will refer to the first and second edition of Kant's *Critique of Pure Reason* in the usual way: A for the first and B for the second followed by the page number. All other references to Kant will be by volume and page number in *Kant's Gesammelte Schriften* (Kant 1902-). I have closely followed the translations of Guyer and Wood (Kant 1998) and Walford (Kant 1992), with occasional modifications.

I have quoted the principles as they appear in the B-edition. The A-edition formulation of the Axioms principle is 'All appearances are, in accordance with their intuition, extensive magnitudes.' The B-edition version more explicitly concerns all determinate intuitions and all mathematical cognition, while the A-edition version is closer to the arguments and summary Kant provides, even in the B-edition.

For grammatical reasons I will refer to the Axioms and the Anticipations in the plural. I am referring, however, to these sections of the *Critique* and not to any set of axioms or 'anticipations.'

experience.[2] The Axioms principle also explains the possibility of any mathematical cognition at all.[3] These principles are central to Kant's theory of human cognition. Nevertheless, I believe that Kant's notions of magnitude, the principles in which they appear, and the arguments for them have been misunderstood. Kant defines the concept of magnitude in the Axioms of Intuition, which also contains the most important claims concerning magnitudes. The focus of this paper will therefore be on the Axioms. It will reconstruct Kant's arguments and analyze his concepts in order to reveal his theory of magnitudes and help explain his theory of human cognition. It will also argue that in Kant's view, mathematical cognition crucially depends on the properties of magnitudes, and that intuition plays an important role in mathematical cognition by allowing us to represent those properties. These results reveal a role for intuition in Kant's philosophy of mathematics that has passed unnoticed, and they open a new line of investigation into Kant's philosophy of mathematics.

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2 Invoking the distinction between pure and applied mathematics can be misleading. Kant does not think of pure mathematics in the way that we might — that is, as a body of knowledge that can be developed without reference to space and time, and whose applicability to space and time is a further question. For Kant, pure mathematics is intimately related to pure space and time. For Kant, the issue of the applicability of mathematics concerns its application to objects of experience appearing in space and time, not its applicability to space and time.

3 The claim that the Axioms principle concerns any mathematical cognition whatsoever may seem surprising if one thinks that the point of the Axioms is limited to arguing for the applicability of mathematics to appearances (cf., for example, Kitcher 1982, §5 and Walsh 1975, 110-1), or that the point of the Axioms is limited to the introduction of a metric into space and time and hence into the field of appearances (Brittan 1978, 92-4 and Guyer 1987, 191). Kant states, however, that the System of Principles contains a systematic presentation of all the synthetic principles of pure understanding, that is, those synthetic principles that flow *a priori* from pure concepts of the understanding under the sensible conditions of the forms of space and time (A136/B175). These would therefore include principles concerning any mathematical cognition whatsoever. Furthermore, Kant calls the principles articulated in the Axioms and the Anticipations mathematical principles. He explains that he does so not because they themselves belong to mathematics, but because they are principles of pure understanding that explain the possibility of the principles of mathematics (B201-2). Kant has in mind principles that explain the possibility of mathematical cognition that includes mathematical principles such as 'two lines do not enclose a space' and '5+7=12.' I defend these claims at greater length in 'The Point of Kant's Axioms of Intuition,' *Pacific Philosophical Quarterly*, forthcoming. The analysis of the argument of the Axioms given below provides further support for this reading.

Section I reconstructs and explains Kant's argument for the Axioms principle. It argues that Kant's argument divides into two stages, the first concerning magnitudes in general and the second concerning extensive magnitudes. Section 2 examines Kant's definition of magnitude, arguing against a modification to it suggested by Hans Vaihinger. Clarifying Kant's definition allows us to identify the fundamental properties of magnitudes. Kant also distinguished between two more specific notions of magnitude, *quantum* and *quantitas*, and Section 3 explains their roles. The results help to explain Kant's theory of magnitudes and their relation to mathematics. Section 4 looks more closely at the second stage of Kant's argument and Kant's conception of extensive magnitude. It uncovers a further fundamental property of magnitudes, one that is important to mathematical cognition.

I The Magnitude Argument

The A-edition version of the Axioms of Intuition begins by explaining the concept of extensive magnitude and argues that all appearances are extensive magnitudes. Kant seems to give further thought to the nature of magnitudes between the two editions of the *Critique* since many of the changes he introduces concern or discuss magnitudes.[4] In the B-edition of the *Critique*, Kant inserts a paragraph at the beginning of the Axioms of Intuition. I quote it and the first sentence of the next paragraph for reference:

> All appearances contain, according to their form, an intuition in space and time, which grounds all of them *a priori*. They cannot be apprehended, therefore, i.e., taken up into empirical consciousness, other than through the synthesis of a manifold through which the representations of a determinate space or time are generated, that is, through the composition of the homogeneous and the consciousness of the synthetic unity of this manifold (homogenous). Now consciousness of a manifold homogeneous in intuition in general, insofar as through it the representation of an object first becomes possible, is the concept of a magnitude (*quanti*). Thus even the perception of an object as appearance is only possible through the same synthetic unity of the manifold of the given sensible intuition through which the unity of the composition of the manifold homogeneous in the concept of **magnitude** is thought; i.e., the appearances are all magnitudes, and indeed **extensive magnitudes**, because as intuitions in space or time they must be represented through the same synthesis as that through which space and time in general are determined.
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> I call an extensive magnitude that in which the representation of the parts makes possible the representation of the whole (and thus necessarily precedes it).... (B202-A167/B203)

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4 See B115, B162, and B201n for examples. Section 26 of the B-deduction (B162), in particular, gives magnitudes a more prominent role.

Commentators who have discussed the added paragraph hold that it either reiterates the A-edition argument or provides a new argument for the principle that all intuitions are extensive magnitudes.[5] However, the passing closing reference to 'extensive magnitudes' notwithstanding, the new paragraph really only concerns magnitudes. It explains the concept of magnitude and argues that all appearances are magnitudes. The explication of extensive magnitudes and the argument that all appearances are extensive magnitudes are left to the second paragraph, that is, the first paragraph of the A-edition. Thus, the B-edition Axioms contain a new argument that supplements the old, and together the new and old arguments constitute a larger two-stage argument.

The two-stage structure of the argument has not been appreciated. This has been, I think, because both arguments turn on the nature of determinate spaces and times, and because the new first paragraph ends with the claim that appearances are magnitudes, and then adds 'and indeed *extensive magnitudes*....' But the argument clearly does not establish that appearances are extensive magnitudes; in fact, extensive magnitude has yet to be defined. The last sentence is intended to point to the A-edition argument, which provides a definition of extensive magnitude and then argues that determinate intuitions and appearances are not only magnitudes but extensive magnitudes in particular. Kant's transitional comment works because the A-edition argument that follows it begins with 'I call an extensive magnitude that in which...' that is, that sort of *magnitude*, which he has just defined and concerning which he has

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5 Kemp-Smith (1979, 394) is representative: he thinks the inserted paragraph needs no special comment after his account of the A-edition argument, though he points out that it gives needed prominence to the role of synthesis. According to H.J. Paton (1965, Vol. II, 111-16), this inserted paragraph contains an argument for the principle of the Axioms that is largely independent of the A-edition argument. Walsh (1975, 113) considers the new paragraph a summary proof that 'simply stands alongside the old,' though it cannot stand on its own. Allison (1983) does not mention the paragraph added in the B-edition; Guyer (1987, 191-2) holds that the paragraph is only a slightly more elaborate argument for the principle.

Some have pointed out that the inserted paragraph ties the Axioms more closely to the transcendental deduction. Wolff (1963, 229) thinks that the purpose of the added paragraph is to ground the A-edition proof in the transcendental deduction. While the paragraph does tie the Axioms more closely to the deduction, that is not its primary purpose. Brittan (1978, 112) does not clearly distinguish the argument of the added paragraph from that of the A-edition, although he claims that it is more closely connected to the Deduction. Longuenesse (1998, 274n70) claims that the paragraph added in the B-edition repeats a portion of the argument in §26 of the B-Deduction. She also states, however, that §26 is itself a repetition of an argument of the A-edition, so it is unclear to me what relationship she thinks the paragraph has to the A-edition argument.

provided an argument. In the A-edition, Kant assumes that the notion of magnitude is sufficiently clear. In the B-edition, he clarifies this notion and provides an additional argument based on it.

The thrust of the added paragraph can be summarized in three steps: 1) The apprehension of an appearance requires a synthesis that generates a determinate space or time, which constitutes a part of the appearance; 2) Every determinate space or time is a magnitude in virtue of the homogeneous manifold that it contains; 3) Therefore, every appearance is a magnitude in virtue of the determinate space or time it contains. This outlines the argument strategy, but a full understanding of Kant's theory of magnitudes turns on the details, so a close reconstruction will be helpful.

1. All appearances contain an intuition in space or time, which grounds all of them *a priori*.[6]

2. Apprehension of appearances thus requires a synthesis of the manifold whereby the representation of a determinate space or time is generated. In other words, appearances, as intuitions in space or time, must be represented through the same synthesis as that through which space and time in general are determined.[7]

3. The synthesis generating [representations of] a determinate space or time requires:
   (a) composition of the homogeneous and
   (b) consciousness of the synthetic unity of this (homogeneous) manifold.

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6 Kant actually states in the first line that all appearances contain an intuition in space *and* time. A paradigm appearance of, say, a table would contain an intuition in both. Kant believes, as he came to emphasize in the B-edition, that our representation of time ultimately depends upon spatial representations, so that a world consisting only of appearances in time would not be possible. However, Kant's claim at the beginning of this argument would exclude the possibility of *any* appearances that occur in time but not space. Since Kant goes on to say that appearances require a determinate space *or* time and ends the argument by referring to appearances insofar as they are intuitions in space *or* time, it seems that he does not wish to limit appearances in this way. On the other hand, Kant did not think an appearance could be in space without also being in time — the condition of all representations.

7 'Appearances as intuitions in space or time' simply refers to appearances insofar as they contain an intuition in space or time, as described in the previous step of the argument.

4. *Definition*: Consciousness of the homogeneous manifold in intuition in general, so far as through it the representation of an object first becomes possible, is the concept of a magnitude (*Quanti*).

5. Therefore, even the perception of an object, as appearance, is only possible through the same synthetic unity of the manifold of the given sensible intuition as that whereby the unity of the composition of the homogeneous manifold in the concept of magnitude [*Quanti*] is thought.

6. Therefore, appearances are all without exception magnitudes [*Quanti*].

This argument rests on implicit premises that can be roughly divided into two classes: those concerning the nature of intuition and those concerning the conditions for the apprehension of appearances. The first class of implicit premises includes the claims that space and time are forms of sensible intuition that impart spatial and temporal properties to both pure and empirical intuitions represented through them, and that all appearances contain an intuition. Kant supports these claims in the Aesthetic.[8] Kant's argument also assumes that every intuition contains a homogeneous manifold, a property that Kant first mentions with respect to magnitudes in a B-edition discussion of the table of categories (B115) and with respect to intuitions in the B-deduction (B162). We will see below that homogeneity is a crucial property of both intuitions and magnitudes.

The second class of implicit premises concerns the conditions for the apprehension of an appearance. These include the claim that the apprehension of an appearance as an object requires a synthetic unity of the manifold in a given intuition. It also includes the claim that apprehending an appearance as an object requires the representation of determinate properties, and that this representation of determinate properties is accomplished through a synthesis of any manifold contained in the appearance. In the case of a spatial or temporal manifold, the resulting representation is of a determinate space or time. The clearest support for these claims is found in the B-deduction.[9]

8 See especially A19-21/B33-5, B41, A26/B42, A34-6/B50-2, and A38-41/B55-58.

9 The culmination of this support is most clearly evident in §26 of the B-Deduction, where Kant states that the synthesis of apprehension requires a unity of the synthesis of the given manifold, and that this unity must be the unity of combination of the manifold in intuition in general according to the categories. Since perception

Determination is at the root of Kant's account of mathematical cognition and magnitudes. According to Kant, lines, surfaces, durations, and the particular spatial and temporal features of an object of experience all count as determinate spaces or times, which we can draw in thought (B137-8, A162-3/B203, A715/B743). Thus, determinate spaces comprise any spatial figure that can be drawn in thought as well as figures constructible in geometry.[10] In §26 of the B-Deduction, Kant states that we, as it were, draw the shape of a house when we apprehend it [zeichne gleichsam seine Gestalt] (B162), suggesting that our representation of the space occupied by a house rests on the synthesis of a determinate space. Step 2 of the Axioms' argument outlined above makes the connection between apprehension of appearances and the generation of determinate spaces and times explicit: the apprehension of an appearance is possible only by means of a synthesis through which the representations of a determinate space or time are generated.

Kant states in the *Prolegomena* that space is something uniform and indeterminate and is the substratum of all determinable intuitions; it is

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itself requires this synthesis according to the categories, Kant concludes that the categories have objective validity for all objects of experience (B161), which is the aim of the deduction.

In the process of establishing the objective validity of the categories, he also establishes important implicit premises of the Axioms' argument. Section §26 of the B-deduction discusses two examples of the necessary use of the categories. The first directly concerns the Axioms. Kant states that in the apprehension of a house, the synthetic unity required in the synthesis of the spatial manifold is the category of magnitude, which he glosses as 'the category of the synthesis of the homogeneous in an intuition in general' (B162). Kant does not further explain the concept of magnitude. That further explanation, and its implication for the possibility of mathematics as well as the applicability of mathematics to experience, constitutes the unique contribution of the Axioms' argument.

Others have noted this connection between the B-deduction and the first paragraph added to the B-edition of the Axioms. As mentioned in n. 5 above, Wolff (1963, 229) thinks that the purpose of the added paragraph is to ground the A-edition proof in the transcendental deduction, and Brittan (1978, 112) notes that the added paragraph is more closely connected to the Deduction. Longuenesse (1998, 243, 274 n. 70) claims both that the passage concerning apprehension of the house is merely an example of what Kant has established and that it contains an argument that Kant repeats in the first paragraph of the Axioms. I read this part of §26 as an example of what Kant has just established, but I do not think that the argument is simply repeated in the Axioms. Rather, it is largely implicitly presupposed in the Axioms, and the Axioms argument builds on it by focusing on the concepts of magnitude and extensive magnitude.

10 I describe Kant's theory of determination in more detail in 'The Point of Kant's Axioms of Intuition,' *Pacific Philosophical Quarterly*, forthcoming.

the understanding that determines space to assume forms such as circles, cones, or spheres. (Ak.4:321)[11] The understanding produces determinate spaces and times through a special synthesis called synthesis of composition, which Kant describes in a note that was also added in the B-edition. He states that the synthesis of composition only generates representations of magnitudes and that it is the 'synthesis of the **homogeneous** in everything that can be considered **mathematically**' (B201n). It is because the representation of determinate spaces and times requires the synthesis of a homogeneous manifold that both they and appearances are magnitudes. Furthermore, that which explains the applicability of mathematics to experience also explains the possibility of mathematics itself. That is, the Axioms' argument establishes the applicability of mathematics to appearance by describing the conditions for mathematical cognition (the representation of determinate spaces and times) and showing that the perception of appearances supposes those conditions. We are now in a position to look more closely at Kant's concept of magnitude.

II Kant's Definition of Magnitude

Kant states that:

> the concept of magnitude (*quantum*) is the consciousness of the homogeneous manifold in intuition in general, so far as through it the representation of an object first becomes possible. (B203)[12]

I will address the nature of a homogeneous manifold in more detail below. Before doing so, however, I would like to discuss an emendation

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11 See also A714/B742.

12 In the German, the passage is 'Nun ist das Bewußtsein des mannigfaltigen Gleichartigen in der Anschauung überhaupt, sofern dadurch die Vorstellung eines Objekts zuerst möglich wird, der Begriff einer Größe (*quanti*).'

Kant uses 'manifold' as an adjective and nominalizes 'homogeneous,' which emphasizes the property of homogeneity. A closer translation of the phrase that preserves this emphasis would be 'manifold homogeneous,' but it is difficult to hear 'homogeneous' as a noun in English. On the other hand, 'manifold of homogeneous elements' suggests a discrete manifold and departs further from Kant's own words. I will therefore revert to 'homogeneous manifold.' The emphasis Kant places on homogeneity is important, however, and should be kept in mind.

I will discuss the significance of restricting the definition of magnitude to *quanta* in the next section.

to the definition proposed by Hans Vaihinger. He suggested adding the phrase 'synthetic unity' to the definition, so that the concept of magnitude is the consciousness of the *synthetic unity* of the homogeneous manifold in intuition. Vaihinger claims that the context compels the addition of 'synthetic unity.' In support he states only that the cause of the omission is apparent to one familiar with the psychology of the typesetter, because these same words are found in the previous sentence; he does not explain which contextual factors he has in mind or how he understands 'synthetic unity.'[13] Nevertheless, influential commentators and translators have followed Vaihinger. His suggestion is noted in Raymund Schmidt's edition of the *Kritik der Reinen Vernunft*, and Norman Kemp Smith adopted the suggestion in his translation of the *Critique*, the standard English translation for most of the twentieth century.[14] Others followed Vaihinger and Kemp Smith in their analyses of the Axioms — for example, H.J. Paton, Ernst Cassirer, and Gordon Brittan.[15]

In order to evaluate Vaihinger's suggestion, we must clarify what sort of unity is meant by 'synthetic unity.'[16] The obvious suggestion is that 'synthetic unity' refers to the category of unity. In that case, the concept of magnitude subsumes the homogeneous manifold in intuition under the category of unity. While this may not have been what Vaihinger had in mind, there are considerations that speak in favor of it. In fact, the relation between the Axioms principle and the categories seems to give quite powerful reasons for adding 'synthetic unity' to the concept of magnitude and understanding it as the category of unity.

The principle of the Axioms in some way corresponds to the conditions imposed by all three categories of quantity — unity, plurality, and allness — under the conditions of sensibility. Immediately following the Table of Categories, Kant states that allness (totality) is the result of a distinct act of the understanding, but that it nevertheless arises from thinking of a plurality as a unity (B111). If we add consciousness of synthetic unity to the concept of magnitude, then the concept of magnitude will require consciousness of a unity of a plurality, i.e. of a homogeneous manifold. Thus, it will require a consciousness of plurality, unity, and allness, and the concept of magnitude will correspond to all three categories of quantity. On the other hand, if we do not add

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13 Vaihinger 1900

14 Schmidt (1926, 217 n. 3), Smith (1965).

15 Paton (1965, Vol. II, 115), Cassirer (1954, 127), and Brittan (1978, 111).

16 I would like to thank an anonymous reviewer for suggesting various improvements to this section.

'synthetic unity,' it is difficult to see how all the categories of quantity will be manifested in a principle about magnitudes.

In this construal of the role of the concept of magnitude, the correspondence between the quantitative categories and the principle of the Axioms is not a perfect one, for the Axioms principle concerns not merely magnitudes but extensive magnitudes (in contrast to the Anticipations principle which concerns intensive magnitudes.) This mismatch speaks against taking the relation between the categories and the concept of magnitude too literally. Nevertheless, Kant gives strong support to this reading of the role of the concept of magnitude when he refers in several passages to 'categories of magnitude' or the 'category of magnitude' (B115, B162, B193, B201). Furthermore, his discussion of the schemata of the quantitative categories is about the schema of the concept of magnitude (A142/B182). It is easy to suppose that the imperfect correspondence is simply a result of the difficulty Kant has in fitting magnitudes into his architectonic scheme.

I think this reason for adding 'synthetic unity' to the definition of magnitude and understanding it as referring to the category of unity has been particularly influential. To anyone reflecting on the relation between the categories and the principle of the Axioms, the addition of 'synthetic unity' to the definition will suggest that the concept of magnitude includes the category of unity. Norman Kemp Smith, who follows Vaihinger's suggestion in his translation of the *Critique*, only briefly discusses the Axioms in his commentary,[17] but he is strongly inclined to see the architectonic as a driving force in Kant's philosophy. H.J. Paton, who also adopts Vaihinger's suggestion, explicitly endorses a correspondence between the categories of quantity and the concept of magnitude when he describes the concept of magnitude as 'the pure category of quantity (totality).'[18] I think, however, that this understanding of Kant's concept of magnitude is mistaken and that it leads to serious misunderstandings of Kant's theory of magnitude.

We have good reason not to include the category of unity in the concept of magnitude and good reason to reject the view that the concept of magnitude corresponds to all three categories of quantity. In Kant's view, the concept of magnitude is not the concept of a totality, that is, a plurality unified under the category of unity. The concept of magnitude is simply the concept of a multiplicity or a manifold, and it is related to the category of plurality rather than all three categories. Concepts such

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17 Smith 1979, 347

18 Paton 1965, Vol. II, 115

as plurality, multiplicity, manifold, and magnitude do not require that their objects fall under the category of unity.

Reflection on the table of categories bears this out. Under the heading of quantity we find the categories of unity, plurality, and allness (or totality) (A80/B106). As mentioned above, Kant states that the last category under each heading derives from a combination of the first two through a distinct act of the understanding, and he asserts that allness is plurality considered as unity (B111). Kant does not elaborate on the nature of this distinct act of the understanding; if, however, the category of plurality required subsuming the plurality under the category of unity, then there would be no difference between the categories of plurality and allness. Thus, the category of plurality does not include the category of unity.

This point is of general importance for concepts of multiplicity, manifold, and other concepts besides magnitude. In Kant's view, to think of something by means of such a concept may require a unity in representing it, but does not entail that we be conscious of a unity in the represented manifold that falls under the category of unity. In other words, the articles in phrases such as 'a multiplicity' or 'the homogeneous manifold' may imply some sort of unity, but not one that falls under the category of unity. I will return to what this non-categorial unity might be below.

There is more evidence that the category of plurality does not require subsuming the plurality under the category of unity in §11 following the Table of Categories. Kant states that the category of allness is not employed to represent the infinite, even though the concepts of unity and multitude, i.e. plurality, can be employed in its representation (B111). I will not attempt to give a proper account of Kant's views on infinity here; it is clear, however, that employment of the category of allness is incompatible with the representation of infinity, while the category of plurality is not.

This point bears directly on the concept of magnitude. Kant states in the Aesthetic that space and time are represented as infinite magnitudes (A25/B39). Since application of the category of allness is incompatible with the infinite, the concept of magnitude includes neither the category of unity applied to a plurality nor the category of allness. The concept of magnitude is closely associated with the category of plurality only.

Section 21 of the *Prolegomena* (AA4:302-4) provides further evidence that the concept of magnitude derives from the category of plurality and does not subsume a homogeneous manifold under the category of unity. Kant presents the table of categories and lists the categories of quantity as shown below:

Transcendental Table
of the Concepts of the Understanding

1.
According to Quantity [Quantität]
Unity (Measure)
Plurality (Magnitude [die Größe])
Allness (Whole)

The concept of magnitude is explicitly associated with the category of plurality rather than with allness.

These texts decisively undermine a reading of the concept of magnitude that includes the category of unity applied to a manifold. This conclusion avoids a distortion of the concept of magnitude that has deleterious consequences for understanding Kant's theory of magnitudes. Furthermore, it entails that the concept of magnitude Kant introduces at the beginning of the Axioms does not correspond to all three categories of quantity after all.

I noted that Vaihinger does not indicate whether he had in mind the category of unity when he made his suggestion; perhaps he had in mind some minimal non-categorial unity. In Kant's view, every concept requires that a manifold be brought to the synthetic unity of apperception, including the concept of magnitude.

In order to evaluate this possibility, it is important to distinguish between the *unity of consciousness* in a concept and the *consciousness of a unity*. The former is a unity in the act of representing; the latter is a unity in that which is represented. According to Kant, the unity of consciousness in a concept finds its source in pure apperception, which brings unity to all our representations.

One might hold that thinking of any multiplicity in one act of consciousness requires not merely a unity in the act of representation but also at least some sort of unity on the side of the represented multiplicity. In the B-deduction, Kant argues that the analytic unity of apperception presupposes a possible synthetic unity of a manifold combined in one consciousness; in fact, this possible synthetic unity is the ground of the identity of apperception.[19] Thus one might think that the unity of consciousness in the concept of magnitude presupposes at least some minimal sort of unity in what is represented. As I mentioned above, even if concepts like magnitude, manifold, multiplicity, and plurality do not require the category of unity, the use of an article in phrases such as 'a manifold' or 'the homogeneous manifold of intuition' might be thought to imply some minimal sort of unity.

In fact, Kant makes a point of distinguishing the synthetic unity thought in a combined manifold from the category of unity. He calls the former a qualitative unity, the latter a quantitative unity (B131). Kant says that we must seek the qualitative unity someplace higher, and what he goes on to say makes it clear that he has in mind the unity of apperception.

Although Kant closely ties qualitative unity to the unity of apperception and hence to the act of representing, he also attributes qualitative unity to the represented. In §12 following the Table of Categories, Kant states:

> In every cognition of an object there is, namely, **unity** of the concept, which one can call **qualitative unity** in so far as by that only the unity of the comprehension of the manifold of cognition is thought, as, say, the unity of the theme in a play, a speech, or a fable. (B114)

In addition, Kant describes qualitative unity as a property of a concept itself, in contrast to its object; it is, he says, a logical criterion of the possibility of a concept (B114-5).

Kant's doctrine of qualitative unity is by no means perspicuous, and I will not attempt to clarify it further. It is sufficient to note that Kant makes room for a qualitative unity that he explicitly distinguishes from the category of unity, and that Vaihinger may have had this sort of unity in mind. But why bother to add 'synthetic unity' at all?

Vaihinger states that context compels its addition. A close reading of the text and the structure of Kant's argument seem to support this claim.

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19 In §16 of the B-Deduction, Kant states:

> … it is only because I can combine a manifold of given representations **in one consciousness** that it is possible for me to represent the **identity of the consciousness in these representations** itself, i.e., the **analytical** unity of apperception is only possible under the presupposition of some **synthetic** one.... Synthetic unity of the manifold of intuition, as given *a priori*, is thus the ground of the identity of apperception itself … (B133-4).

This synthetic unity is the unity presupposed by the combination of any manifold in one consciousness.

Step 3(b) of our reconstruction states that generating the determinate space or time of an appearance requires 'consciousness of the *synthetic unity* of a homogeneous manifold.'[20] Step 4 gives the definition of the concept of magnitude as 'consciousness of a homogeneous manifold,' and step 5 infers that the concept of magnitude is required for any appearance. If we do not add 'synthetic unity' to the definition of magnitude, then step 3(b) will require something distinct from the concept of magnitude given in step 4, and, strictly speaking, the argument will not be valid. Furthermore, in step 5, Kant refers to thinking of 'the unity of the combination of the homogeneous manifold in the concept of magnitude.' He therefore seems to think that unity is part of the concept.

These considerations make a straightforward appeal to nothing more than the logical form of the argument and a close reading of the text; they do not depend on what sort of unity is meant by 'synthetic unity.' They could therefore have prompted Vaihinger to add 'synthetic unity,' where he took that to mean a merely qualitative non-categorial unity.

Nevertheless, I do not think that these considerations hold up to scrutiny. First, Kant might appear to include unity in his reference to the concept of magnitude in step 5, since he refers to 'that whereby the unity of the composition of the homogeneous manifold in the concept of magnitude is thought.' Nevertheless, the phrase 'in the concept of magnitude' can be read as modifying 'homogeneous manifold' and not 'the unity of the composition.'[21]

Second, a closer look at the content of the argument actually speaks against adding 'synthetic unity' at all. Kant's argument of the Axioms unfolds the requirements of apprehending an appearance, or, as he also puts it, the perception of an object as an appearance. Step 2 asserts that apprehension or perception of an object as appearance requires the generation of the representation of a determinate space or time. In Kant's view, the synthesis generating a determinate space or time is governed by concepts; moreover, the representation of a *determinate* space or time requires a completion in the synthesis, a completion that is subsumed under the category of unity and hence totality or allness. Thus, when Kant asserts in step 3 that generating the representation of a *determinate* space or time requires both (a) composition of the homogeneous and (b)

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20 See the argument reconstruction on pp. 415-16 above.

21 In German, the passage is 'Also ist selbst die Wahrnehmung eines Objekts, als Erscheinung, nur durch dieselbe synthetische Einheit des Mannigfaltigen der gegebenen sinnlichen Anschauung möglich, wodurch die Einheit der Zusammensetzung des mannigfaltigen Gleichartigen im Begriffe einer Größe gedacht wird.'

consciousness of the synthetic unity of this homogeneous manifold, he most likely has in mind the category of unity.

Nevertheless, this result does not compel us to add the category of unity to the concept of magnitude in order to preserve the validity of the argument. The consciousness required for generating determinate spaces and times is a consciousness through concepts of the understanding. The consciousness of the synthetic unity of a homogeneous manifold required for generating determinate spaces and times *includes within it* a consciousness of a homogeneous manifold. The consciousness required for generating a representation of determinate space or time for an object includes much more than what belongs to the concept of magnitude, but it necessarily includes what belongs to the concept of magnitude.[22] In other words, one cannot be conscious of the synthetic unity of a homogeneous manifold as a synthetic unity of a homogeneous manifold without being conscious of the homogeneous manifold. It is this latter consciousness through concepts that Kant defines as the concept of magnitude.

One could make this implicit premise explicit by adding the following step:

> 3.5 A consciousness through concepts of the synthetic unity of a homogeneous manifold includes a consciousness through concepts of the homogeneous manifold.

Inserting this step into the argument would make the inference strictly valid without requiring the addition of 'synthetic unity' to the definition of magnitude.

One might still argue that the concept of magnitude requires a qualitative unity, and hence that it is more exact to include 'synthetic unity' in the concept of magnitude. However, inserting 'synthetic unity' does not bring any apparent clarity, and it invites confusion with the category of unity. In fact, I think this is exactly the effect it has had on some commentators. Moreover, adding synthetic unity needlessly complicates the definition and draws attention away from the fact that the definition of magnitude focuses on the properties of being a manifold

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22 Kant's definition states that the concept of magnitude is the consciousness of a homogeneous manifold in intuition in general, 'so far as through it the representation of an object first becomes possible.' I do not think that this is a restrictive clause; Kant is simply emphasizing that he is considering the concept of magnitude in so far as what it represents is required for the apprehension of appearances. (I have not been able to find any occurrences in Kant of clauses introduced by 'so far as' [so fern] that are unequivocally restrictive.)

and being homogeneous. We should therefore accept Kant's formulation of the definition of magnitude as his considered view of the concept during the critical period.[23]

Leaving 'synthetic unity' out of the definition of magnitude narrows down the characteristic properties of being a magnitude. Kant defines magnitude as a homogeneous manifold in intuition in general, thereby abstracting from our particular human forms of intuition, space, and time. Whether or not there are other forms of intuition is not something Kant thinks we can know, but he chooses not to restrict his definition to our forms of intuition. This abstract characterization is also present in the house example of §26 of the B-Deduction mentioned earlier. It states that if we abstract from the form of space, we arrive at the 'category of the synthesis of the homogeneous manifold in intuition in general, that is, the category of magnitude' (B162). This suggests that features of any intuition in general, rather than the particular features of space and time, are important for cognition and especially for mathematical cognition. I will return to this point below.

Kant does not use the term 'manifold' with a specific technical meaning in mind; he uses it for any kind of multiplicity whatsoever. The key property of magnitudes, then, is being a *homogeneous* manifold.[24] Furthermore, it is a homogeneous manifold *in intuition*: without 'synthetic unity' added to the concept of magnitude, we can see that a magnitude does not presuppose anything but a plurality in intuition that is homogeneous.

This analysis of the concept of magnitude leaves us with a puzzle and some unanswered questions. It does not explain how the employment of the categories of quantity is accounted for by the principle of the Axioms or why Kant sometimes refers to the category or categories of magnitude. We also do not yet have an account of how the concept of magnitude contributes to the mathematical determination of spaces and times at the root of mathematical cognition. The answer to these questions turns on Kant's distinction between two notions of magnitude: *quantum* and *quantitas*.

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23 Kant added discussions of magnitude both to the B-deduction and to the paragraph added in the B-edition Axioms. Furthermore, all the references to the category or categories of magnitude only occur in the B-edition. Thus, Kant seems to have given further thought to magnitudes between the two editions; it is his considered view in and after the B-edition with which I am most concerned.

24 As noted in footnote 12 above, a more literal translation of Kant's definition states that a magnitude is a 'manifold homogeneous' in intuition in general, a formulation that emphasizes the role of homogeneity.

III *Quantum* and *Quantitas*

Kant's definition of magnitude is, as his parenthetical insertion indicates, the definition of a *quantum* in particular. Hence, all that we have discussed up to this point concerns *quanta*. Later in the Axioms of Intuition, Kant draws a distinction between a *quantum* and a *quantitas*. The Latin *-itas* ending indicates an abstract entity or property. This leads one to expect that *quantum* refers to something relatively concrete, something that is a magnitude, while *quantitas* refers to the magnitude of a thing in some more abstract sense.[25] Kant uses the term magnitude [Größe] for both *quantum* and *quantitas*, adding these terms in parentheses when he chooses to be careful. He is not always careful, however. As noted above, Kant refers to the categories and even the category of magnitude rather than to the categories of quantity, and he does not specify which sense of magnitude he has in mind.

A *quantum* is more concrete than a *quantitas*. Being relatively concrete, however, does not itself entail that a *quantum* has properties particular to space or time. The only human forms of intuition are space and time; this fact together with the definition of a *quantum* entails that for humans, concrete *quanta* represented in intuition will have spatial or temporal properties. As noted in the previous section, however, Kant defines *quantum* as a homogeneous manifold in intuition *in general*, a definition that abstracts from our particular human forms of intuition, space, and time.[26] This point is important because it suggests that one role for intuition in cognition, especially mathematical cognition, is simply to represent a homogeneous manifold. This is a property common to both space and time, and hence does not depend upon the more particular properties of either.

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25 There is no similar distinction between the concrete and abstract form of the German term for magnitude [Größe]. Kant uses the term quantity [Quantität] for the quantitative forms of judgment and the categories of quantity, but he declines to use the terms 'magnitude' and 'quantity' to indicate the distinction he has in mind.

26 This feature of *quanta* might be overlooked if the use of the expression 'in general' in the definition of *quantum* is taken to suggest something abstract. The German term is 'überhaupt' rather than 'allgemein,' a term Kant also uses on occasion but not in the present context. 'Überhaupt' carries a connotation of 'at all' and need not mean that a *quantum* itself is something general or abstract. For example, reference to a 'horse in general' might suggest something abstract in a way that 'any horse at all' or 'any horse whatsoever' does not. A more accurate translation might therefore be: a *quantum* is a manifold homogeneous in any form of intuition at all.

Although Kant defines a *quantum* in the paragraph added to the Axioms, he nowhere defines *quantitas*, and determining its nature is difficult. The Axioms, the Discipline of Pure Reason, and related texts help explain the nature of *quanta* and *quantitas* and how mathematics relates to them. After examining these texts, I will turn to the Schematism, which helps uncover the relation between *quanta* and *quantitas*.

In the Axioms, Kant characterizes *quantitas* as that which answers the question 'How big is something?' — for example, 'How big is the notebook on my table?'[27] Kant states that some mathematical truths concern *quanta*, but not *quantitas*. He says, for instance, that strictly speaking the axioms 'Between two points only one straight line can be drawn' and 'Two straight lines cannot enclose a space' only relate to magnitudes (*quanta*) as such. (A163/B204). These axioms concern spatial magnitudes, but not their amounts, implying that a *quantitas* is an amount. A paradigmatic answer to the question 'How big is something?' and hence what its *quantitas* is requires a measurement — that is, a specification of a unit of measure and a number that specifies how many units are equivalent to that which is measured. Nevertheless, the amount of something is independent of the unit chosen and is a property of the object that is measured: the length of a book, for example, is the same whether we describe it as 11 inches or 28 centimeters.

For all that has been said up to this point, a *quantitas* could be the abstract particular amount of a concrete particular *quantum* — this (and only this) book's length, for example. The book might have a *quantitas* that is in some way equivalent to the *quantitas* of this sheet of paper, but each would have its own *quantitas*. Alternatively, a *quantitas* might be the abstract amount shared in common by different *quanta*: the *quantitas* of this book might be one and the same *quantitas* as this sheet of paper.

In other passages, however, Kant suggests a broader understanding of *quantitas* restricted to neither particular or common abstract amounts of *quanta*. He claims, for example, that statements of numerical relation such as 7+5=12 are about *quantitas* (B204-5). On the face of it, such statements are most naturally understood as about numbers rather than as covert references to amounts of *quanta*; they are not obviously about how big some concrete thing is, or how many there are of some concrete thing. This at least suggests that Kant employed a broader notion of *quantitas* that includes numbers apart from the things numbered.

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27 The term for 'big' in German has the same root as magnitude, a fact to which Kant seems to call attention: 'Was aber die Größe, *(quantitas)* d. i. die Antwort auf die Frage: wie groß etwas sei? betrifft...' (A163/B204).

Kant further contrasts *quanta* and *quantitas* in the Discipline of Pure Reason, where he states:

> … mathematics does not construct mere magnitudes (*quanta*) as in geometry; it also constructs mere magnitudes (*quantitas*), as in algebra. In this it wholly abstracts from the properties of the object [Gegenstand] that is to be thought in terms of such a concept of magnitude (A717/B745).

Wholly abstracting from the properties of an object suggests that the concrete object has been left entirely behind. This leaves open the possibility that *quantitas* (or at least the *quantitas* of algebra) has no direct connection to *quanta*, which would be in keeping, at least in this respect, with an abstract conception of *quantitas* that includes numbers. In one such view, the *quantitas* of arithmetic refer to particular numbers, such as 5 or 7, while the *quantitas* of algebra abstract from the properties of particular numbers, referring to numbers in general by means of variables. Alternatively, Kant's emphasis on the completeness of abstraction might lead one to think that algebra has no object at all, or has as its object a thing in general.[28] For many early modern mathematicians, algebra was viewed more as a problem-solving technique than its own discipline, which lends this latter view historical plausibility.

Nevertheless, in this passage, the *quantitas* of algebra is described as that concept by means of which an object is thought; it is *not* described as even indirectly about numbers. The contrast to *quanta* in the same sentence suggests that the objects whose properties are abstracted from are *quanta* like those found in geometry. Moreover, early modern mathematicians applied algebra widely to geometrical problems, and in their interpretation of this application, the objects referred to seem to be geometrical *quanta*.[29] Finally, Kant states that algebra abstracts wholly from the properties of objects, but if it truly abstracts from *every* property, it is unclear how the concept of *quantitas* can relate to objects at all by means of them. What, in Kant's view, are the objects of mathematics?

Several texts outside of the *Critique* shed light on what Kant had in mind. In §4 of the *Inquiry*, Kant states 'The object of mathematics is magnitude' (2:282). He adds that algebra is the general doctrine of magnitudes (2:282). This pre-critical view is echoed in a 1789 letter to Carl Rheinhold, which states that algebra concerns mere magnitudes

28 See, for example, Parsons (1983b, 134-5) and Friedman (1992, 112-22, especially 113, 114, and 122; but see also 114, fn34).

29 Lisa Shabel has argued that Kant's conception of algebra should be understood as intimately connected to geometry. See Shabel (1998).

without qualities. Algebra therefore abstracts from qualities, but not from the property of being a magnitude. By doing so, algebra focuses on the ratios between magnitudes (11:42). Kant also describes algebra as a general doctrine of magnitudes in a letter to Schultz of 1788.[30] It is therefore likely that when Kant states that 'algebra wholly abstracts from the constitution of the object that is to be thought in accordance with the concept of magnitude,' he does not intend that we abstract from those properties that make the object a magnitude.

In this interpretation, algebra considers *quanta* merely in so far as they are homogeneous manifolds in intuition in general. When algebra is applied to geometry, for example, it only considers geometrical *quanta* insofar as they are homogeneous manifolds that can be combined in various ways; it abstracts from the particular spatial properties of geometrical *quanta* — their relative positions, for example — and solves the geometrical problem without recourse to geometrical constructions.[31] When algebra is applied to discrete *quanta*, it only considers them insofar as they are homogeneous manifolds that can be counted and combined into totals, and here too, algebra abstracts from their particular properties. Mathematics is about magnitudes, and mathematical objects are *quanta*. This places Kant's theory of magnitudes at the heart of his theory of cognition, as one would expect given the centrality of the Axioms and Anticipations principles. In particular, it places Kant's theory of magnitudes at the heart of his theory of mathematical cognition, and hence his philosophy of mathematics.

The Schematism provides further clues into the relations among *quanta*, *quantitas*, the categories of quantity, and the nature of mathematical determination. According to Kant, schemata are representations that

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30 Kant also indicates in the *Metaphysical Foundations of Science* that *mathesis* is a general doctrine of magnitude (4:489), while in his mathematics lectures, he calls algebra a *Mathesis Universalis* (29:49).

31 Furthermore, discrete magnitudes, the sorts of *quanta* that can be counted, are also treated insofar as they are homogeneous manifolds in intuition. Obviously, more needs to be said to make good on these claims. That the objects of mathematics are magnitudes and that homogeneous manifolds are crucial to Kant's philosophy of mathematics derives strong further support from Kant's views on the relations among algebra, magnitudes, and the Eudoxian theory of proportions. For more on the role of homogeneous manifolds in Kant's philosophy of mathematics, see my 'Kant's Philosophy of Mathematics and the Greek Mathematical Tradition,' *Philosophical Review*, forthcoming. For more on discrete magnitudes, arithmetic and algebra in particular, see my 'Kant on Arithmetic, Algebra, and the Theory of Proportions,' *Journal of the History of Philosophy*, also forthcoming.

mediate concepts and intuitions. More specifically, schemata are 'rules for the determination of our intuition in accordance with a certain general concept' (A141/B180). This is the most general description of their role, but schemata play different roles for pure concepts of the understanding on the one hand, and empirical and pure sensible concepts on the other. Kant holds that pure concepts of the understanding, in contrast to empirical concepts and pure sensible concepts like the concept of a triangle, have nothing in their content in common with intuition. For that reason, pure concepts of the understanding cannot stand in direct relation to intuitions. The categories are pure concepts of the understanding, and schemata have a special role to play in mediating between them and intuitions. The schemata of the categories take the form of rules for determining the time relations among representations. In Kant's account, concepts are rules of the understanding, and in so far as schemata represent rules, they share with concepts an expression of generality. As determinations of time, schemata relate to every representation that appears to us in time and hence everything that appears to us in intuition. By relating to both categories and intuitions in this way, schemata are meant to bridge the gap between them.[32]

Like the schemata for the categories, the schemata for empirical concepts and pure concepts of sensibility are 'rules for the determination of our intuition in accordance with a certain general concept' (A141/B180). There is an important difference, however. The schemata of pure concepts of the understanding can never be brought into an image [in gar kein Bild gebracht werden kann] (A142/B181), while a schema of an empirical or pure concept of sensibility is 'a representation of a general procedure of imagination for providing a concept with its image [Bild]' (A140/B179-80).

Kant states that the schema of a pure sensible concept, e.g., the concept of a triangle, indicates 'a rule of the synthesis of the imagination in regard to pure shapes in space' (A140-1/B179-80). A schema represents a general procedure for providing an image for a concept and expresses a generality that allows us to reason about all the objects that fall under a concept. Kant holds that schemata thereby avoid the problems posed by a theory of reasoning that treats ideas as particular images. For example, reasoning by means of a schema is supposed to permit us to draw

32 My aim is not to explain or evaluate Kant's views in the Schematism, which is well beyond the scope of this paper; my aim is to explain enough to draw out a few important points concerning *quanta* and *quantitas*.

conclusions about all triangles (or all triangles of a particular type) rather than just one particular triangle.[33]

Kant describes the schemata of the various categories. He begins with the categories of quantity, but rather than treat the concepts of unity, plurality, and allness separately, he introduces the concept of magnitude:

> The pure image of all magnitudes (*quantorum*) for outer sense is space; that of all objects of the senses in general is time. The pure **schema of magnitude** however (*quantitatis*), as a concept of the understanding, is **number**, a representation which collects [zusammenbefaßt] the successive addition of one to one (homogeneous). Number is thus nothing other than the unity of the synthesis of the manifold of a homogeneous intuition in general, brought about through my producing time itself in the apprehension of the intuition. (A142-3/B182)

It is clear that the concept of magnitude is in some way intended to play the role of all three quantitative categories — unity, plurality, and totality. It is also clear that the schema corresponding to these categories is number, an important claim for Kant's philosophy of mathematics. The relation between *quanta* and *quantitas* is less clear. Since the concept of magnitude stands in for the three categories and Kant contrasts the image and the schema of a concept, one might think that a *quantum* and *quantitas* are the image and schema of the concept of magnitude, respectively.[34] However, both '*quantum*' and '*quantitas*' appear in the genitive,

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33 Kant also provides a less perspicuous example from arithmetic. He states that five dots in a row provide an *image* of the number five. In contrast, the *schema* for a concept of a number such as five is the representation of a general procedure of the imagination, the representation of a method for representing a multitude. Kant states:

> If I only think a number in general, which could be five or a hundred, this thinking is more the representation of a method for representing a multitude [Menge] (e.g. a thousand) in accordance with a certain concept than the image itself, which in this case I could survey and compare with the concept only with difficulty (A140/B179).

Kant either means that there is a unique schema for each individual number, whether it be five, one hundred, or one thousand, or he means that there is one schema for the generation of all numbers and that the concept of five, say, picks out the multitude generated by five iterations of this general procedure. Parsons makes this point (Parsons 1984). In either case, a schema allows us to generate representations of multitudes. See footnote 31 above.

34 This interpretation is also suggested by the placement of 'however' in the description of the schema, which separates 'schema of magnitude' from *quantitas* in a way suggesting that the former is the topic of the sentence and is equated with the latter. The German is 'Das reine *Schema* der *Größe* aber (*quantitatis)*, als eines Begriffs des Verstandes …'

so that Kant is describing the image of *quanta* in the first case and the schema of *quantitas* in the second, that is, the image and schema of different concepts. The relationship between *quanta* and *quantitas* is nevertheless close enough that the image of *quanta* on the one hand and the schema of *quantitas* on the other fill out Kant's account of magnitude. This closeness is reinforced by Kant's habit of referring to one or both *quanta* and *quantitas* simply as 'magnitude.'

The passage above also reveals that the concept of *quantitas*, not of a *quantum*, represents the categories. Kant is giving an account of how the categories of quantity are applied by means of their schema, and his explanation describes the schema of *quantitas*. On the other hand, his reference to *quanta* is limited to its image, and he has stated that the schema of a category cannot be brought into an image. In addition, Kant follows the phrase 'schema of magnitude (*quantitas*)' with 'as a concept of the understanding,' indicating that magnitude as *quantitas* is treated as a concept of the understanding. Thus, *quantitas* corresponds to the categories of quantity.[35]

If the concept of *quantitas* corresponds to the categories of quantity, it should in some way involve the category of allness as well as unity and plurality. As mentioned above, Kant states that the schema of *quantitas*, as a concept of the understanding, is number. Kant shares the view widely held in the early modern period that numbers must be finite; this restriction is reflected in Kant's claim that number requires an application of the category of allness. In a passage referred to earlier following the Table of Categories, Kant states that the category of allness is a distinct act of the understanding in which a plurality is thought of as a unity. In emphasizing that it is a distinct act, Kant states:

> The concept of a *number* (which belongs to the category of allness) is not always possible where the concepts of plurality and unity are (for instance, in the representation of the infinite) … (B111).

Kant's point is that we may think of a plurality of unities whose synthesis never reaches completion. Kant defines what he calls the 'true transcendental concept' of the infinite as that whose 'successive synthesis of the unity in the measurement of a *quantum* can never be completed'

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35 In addition, as noted, Kant claims that the schema of *quantitas* is number. At the conclusion of the Schematism, Kant describes number as *quantitas phenomenon* and asserts that the schema is the phenomenon, or the sensible concept of an object in agreement with the category (A146/B186). Since the schema of *quantitas* is a schema in agreement with the category, *quantitas* is equated with the categories of quantity.

(A432/B460).[36] In contrast, number requires a completion of the successive synthesis that is expressed by the category of allness. Hence, the schema of *quantitas* requires the concept of allness, and *quantitas* plays the role of all three categories of quantity.[37]

The role of *quantitas* contrasts with that of *quanta*. As noted, Kant holds that no image can be provided for the categories. Thus the image of *quanta*, that is, space and time, cannot themselves provide an image for the categories of quantity. Instead, space and time provide a homogeneous manifold in intuition, which is determined in accordance with the categories of quantity. It is significant in this regard that Kant does not refer to the image of a particular *quantum*; he refers to the image of *all quanta* of outer sense and *all* objects of inner sense. The images to which Kant refers to are not particular determinate spaces and times but space and time themselves — that is, undetermined space and time as given intuitions. Kant thereby emphasizes that '*quanta*' refers to any homogeneous manifold in intuition in general, whether or not that manifold has been determined.

In fact, Kant elsewhere explicitly allows that we can intuit indeterminate *quanta* (A426/B454). In the Transcendental Aesthetic, moreover, he makes it clear that space and time as given intuitions, and hence as undetermined, are magnitudes (A25, B39). These considerations show that the concept of *quanta* applies to any homogeneous manifold in intuition in general, whether or not it has been determined. In contrast, the schema of *quantitas* determines *quanta* and thereby generates the determinate spaces and times underlying both mathematics and appearances.[38] Space and time as indeterminate *quanta* serve as the substrate

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36 I take this definition to be Kant's own, not one presented as the views of the imagined defenders of the thesis or antithesis. The definition occurs in the Remarks on the First Antinomy rather than in the formulation of the Antinomies themselves. It is also presented as the true *transcendental* concept of infinity, which neither the defender of the thesis nor the antithesis could be expected to employ.

37 These results allow a further clarification of the principle of the Axioms. The principle asserts that all appearances *are*, as regards their intuition, extensive magnitudes. A *quantitas* is an answer to the question of how big something is, and is about an amount. An appearance is something that *has* an amount, but it contains a homogeneous manifold in its intuition. So when Kant says that appearances *are* extensive magnitudes, he must mean *quanta*. A clearer version of the principle of the Axioms would therefore be: All appearances, as regard their determinate intuition, are extensive *quanta*. A corresponding clearer version of the B-edition principle would be: All determinate intuitions are extensive *quanta*.

38 The schema of *quantitas* is number, which is a 'representation which collects the successive addition of one to one (homogeneous)' (A142/B182). It is this which

upon which the understanding operates by means of the schema of *quantitas*.[39]

The foregoing analysis makes it apparent that the notion of magnitude plays a central role in Kant's theory of cognition — in particular, his theory of mathematical cognition. Kant holds that mathematics is about magnitudes and that the objects of mathematics are magnitudes. Moreover, Kant calls the principles of the Axioms and Anticipations mathematical principles because they explain the possibility of mathematics as well as the application of mathematics to appearances. These principles concern a special mathematical synthesis called composition, which only synthesizes a homogeneous manifold in intuition. Homogeneity therefore plays a central role in Kant's philosophy of mathematics. The fact that *quanta* include undetermined space and time and that they are determined by the schema of *quantitas* shows how closely Kant's notion of *quanta* is tied to intuition. Furthermore, the definition of *quanta* as a homogeneous manifold in intuition in general underscores the fact that what is of relevance for Kant's philosophy of mathematics is the homogeneity of intuition in general, and not the more particular properties of space or time. These considerations strongly indicate that intuition plays an important role in Kant's philosophy of mathematics by allowing the representation of a homogeneous manifold.

IV *Quanta, Quantitas*, and Extensive Magnitude

Although we have uncovered the key properties of *quanta*, Kant has more to say about magnitudes. The Axioms and the Anticipations concern extensive and intensive magnitudes, respectively. As mentioned earlier, a circle or the space occupied by the book on my table are paradigm extensive magnitudes; the intensity of a light is a paradigm

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serves as a rule or method for producing a determinate space or time in accordance with the categories of quantity. Exactly *how* number determines discrete and continuous magnitudes is an important issue, but it is beyond the scope of this paper. I say more of relevance on this issue in 'Kant on Arithmetic, Algebra, and the Theory of Proportions,' *Journal of the History of Philosophy*, forthcoming. Kant's understanding of number is further complicated by possible shifts in his views before and after writing the *Critique*; see Parsons (1984).

39 See 4:321-2 for a characterization of intuition as indeterminate substratum. For more on the contrast between space and time as undetermined intuition and determinate spaces and times, see my 'The point of the Axioms of Intuition,' *Pacific Philosophical Quarterly*, forthcoming.

intensive magnitude.[40] The Anticipations establish that sensations and that which is represented in the appearances corresponding to them (the 'real') are intensive magnitudes (A165/B207). The Anticipations explain the possibility of applying mathematics to phenomena that have intensity. There is no indication that intensive magnitudes play a role in explaining the possibility of mathematics apart from the application of mathematics to appearances. In contrast, the properties of extensive magnitudes do play a role in explaining mathematical cognition, which gives extensive magnitudes a priority over intensive magnitudes. In fact, we cannot even recognize an intensive magnitude as a magnitude at all without the aid of determinate times, which are extensive magnitudes (A166-7/B207-9). Without the categories of quantity and the corresponding concept of *quantitas* there would be no representation of determinate magnitudes whatsoever, no theory of magnitude, and no possibility of mathematics or its application.[41]

In the second-paragraph of the B-edition Axioms, Kant defines an extensive magnitude as one in which 'the representation of the parts makes possible the representation of the whole (and therefore necessarily precedes the latter)' (A162/B203). Kant's strategy is to argue that determinate spaces or times are extensive magnitudes, that all appearances contain determinate spaces or times, and hence that all appearances are extensive magnitudes. The argument can be reconstructed as follows:

1. An extensive magnitude is that in which the representation of the parts makes possible the representation of the whole (and therefore necessarily precedes the latter).

2. The representation of a determinate space or time is only possible through a successive synthesis from part to part.

3. Anything cognized by means of a successive synthesis requires that the representation of the parts makes possible the representation of the whole.[42]

40 Kant's notion of extensive magnitude therefore does not correspond to a modern conception of what can be extensively measured. For more on the modern conception of measurement, see Krantz (1971, Vol. 1, chs. 1 and 3).

41 This emphasis on the role of extensive magnitudes is reinforced by Kant's claim about the principle of the Axioms: 'This transcendental principle of the mathematics of appearances greatly enlarges our *a priori* knowledge. For it is that *alone* which can make pure mathematics, in its complete precision, applicable to objects of experience' [my italics] (A165/B206).

4. Therefore, all determinate spaces and times are extensive magnitudes.

5. All appearances contain an intuition in space and time that is determined.

6. Therefore, all appearances are extensive magnitudes.[43]

This argument, together with that added at the beginning of the Axioms, completes the two-stage argument that all appearances are extensive magnitudes. The crucial step of this stage states that the representation of a determinate space or time is only possible through a *successive* synthesis. This step is supported by the Schematism doctrines that we have already examined: the determination of intuition in accordance with the categories of quantity is guided by the schema of *quantitas*, i.e. number, which is a 'representation that summarizes the *successive* addition of one to one (homogeneous)' (A142/B182, my italics). Kant holds that the addition must be successive because the schemata mediate the categories and intuitions through a transcendental determination of time.

There is an important contrast between the argument added to the B-edition we considered earlier and this one. The former does not mention successiveness at all and turns on the general conditions for

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42 One might ask which parts are presupposed and in which sense they must be represented. There are really two issues here. First, a whole line segment, for example, can be divided into two line segments indefinitely many ways. Is each of these pairs of parts to be represented, and in what sense? Second, any line segment that forms a part of a line is itself a whole line. If it is an extensive magnitude, this will in turn require a representation of its parts. What will stop a regress? Kant is not, however, claiming that the presupposed parts are actually apprehended as parts. Kant's point is that we cannot apprehend a line, for example, without apprehending it as having distinguishable parts, even if we do not explicitly cognize or otherwise pick them out.

43 This reconstruction shows that the argument can only establish a claim about determinate intuitions and appearances; it cannot establish that all intuitions, in particular space and time as given intuitions, are extensive magnitudes. A clearer statement of the A-edition principles would therefore be: All appearances are, as regards their *determinate* intuition, extensive *quanta*. A corresponding clearer statement of the B-edition formulation would be: All *determinate* intuitions are extensive *quanta*. Elsewhere, I have defended this interpretation of the Axioms principle by appealing to the role of the Axioms in the *Critique*; see 'The Point of Kant's Axioms of Intuition,' *Pacific Philosophical Quarterly*, forthcoming. The present analysis of the argument lends this interpretation further weight.

determining any homogeneous manifold in intuition in general. The latter concerns a particular feature of both space and time: they can only be represented through a successive synthesis, a synthesis which itself determines time.[44] At this stage of the argument, Kant is appealing to the particular properties of space and time and not merely to the properties of any homogeneous manifold in intuition in general.

The importance of the successiveness of this synthesis has been noted. Charles Parsons has argued, for example, that in Kant's account the successive construction or apprehension of marks or signs provides a model for the structure of the numbers.[45] Michael Friedman has argued that the successiveness of time makes it possible to represent the unlimited iterability of mathematical operations such as the construction of a line or the addition of a number.[46] It is clear that successive synthesis in time plays an important role in Kant's philosophy of mathematics. Kant also holds, however, that the successive synthesis requires that the representation of the parts makes possible and precedes the representation of their wholes. I do not think that the part-whole relations to which Kant appeals have been sufficiently appreciated.[47]

Kant emphasizes the importance of this property in the Axioms. He asserts that successive synthesis guarantees that all appearances are represented as aggregates, that is, collections of previously given parts (B204). Furthermore, they are represented as composed [zusammengesetzt] of these parts. I noted earlier that Kant identifies a special synthesis of composition that only operates on magnitudes. Kant further distinguishes between the synthesis of composition of extensive and intensive magnitudes. He calls the former the synthesis of aggregation (B201n). Successive synthesis entails that determinate spaces and times, and hence appearances, are represented as aggregates of previously given parts out of which they are composed. I will call this the part/whole

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44 As Kant puts it, the successive addition of the schema of *quantitas* takes place through 'a generation of time itself in the apprehension of an intuition' (A142-3/B182), which is an *a priori* time-determination of the time-series [Zeitreihe] (A184-5/B185).

45 Parsons (1983b), especially 139-41.

46 Friedman (1992), chs. 1 and 2; see especially 63, 87-9, 92, 116-20. Friedman further develops his view to allow for a role of intuition in allowing us to represent the translation and rotation of a point of view (Friedman 2000, 191-3).

47 Kirk Dallas Wilson (1975) and Gordon Brittan (1978) have also emphasized the importance of part-whole relations in Kant's conception of intuition.

aggregation property. Immediately following his assertion that appearances are intuited as aggregates, Kant adds:

> This successive synthesis of the productive imagination in the production of figures is the foundation on which the mathematics of extension (geometry) with its axioms is grounded. It expresses the conditions of a sensible intuition *a priori*, under which alone the schema of a pure concept of outer appearance can arise: e.g., between two points only one line is possible, two straight lines do not enclose a space, etc. These are axioms that actually only concern magnitudes (*quanta*) as such (B204).

Kant indicates in this passage that successiveness plays an important role in geometry. Immediately prior to this passage, Kant states that the successive synthesis is responsible for our representing determinate spaces and times as aggregates of antecedently given parts. What is crucial to geometry is not merely the successiveness of the synthesis, but that such synthesis leads to a representation of the part-whole structure of determinate spaces and times. Consequently, part-whole relations, as well as homogeneity, appear to play a central role in Kant's theory of magnitudes and mathematical cognition.

V Conclusion

I have argued that Kant's theory of magnitudes is of far greater interest and importance than is usually supposed. Kant has well-developed views concerning magnitudes, views that he further develops between the two editions of the *Critique*. I have suggested various improvements in our understanding of Kant's treatment of magnitudes, in particular, that the Axioms of Intuition argument has two-stages and that Kant's definition of magnitude should not be amended as some commentators have suggested. I have also argued that *quanta* include undetermined magnitudes and that the notion of *quantitas* corresponds to the quantitative categories. While there are still many textual questions to be answered, I hope these suggestions have improved our understanding of Kant's *Critique*. I also hope to have shown that Kant's theory of mathematical cognition rests on his theory of magnitudes and that in his view mathematics is about magnitudes. In addition, Kant holds that the homogeneity of magnitudes is crucial for mathematical cognition and that intuition is required to represent a homogeneous manifold — a role for intuition in Kant's philosophy of mathematics that has not been appreciated. Finally, part-whole relations of magnitudes also play an important role in Kant's philosophy of mathematics. Why Kant thinks that intuition is required to represent homogeneity and how part-whole relations figure in mathematical cognition will require further investigation. I hope in this paper to have provided a convincing case that Kant's theory of magnitudes is central to Kant's critical philosophy, and thereby hope to have opened an important new avenue of research into Kant's philosophy of mathematics.[48]

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48 I pursue these issues further in 'Kant's Philosophy of Mathematics and the Greek Mathematical Tradition,' *Philosophical Review*, forthcoming.

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