

Paradox Lost

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Frederic Fitch's celebrated reasoning to the conclusion that all truths are known¹ can be interpreted as a *reductio* of the claim that all truths are knowable. Given this, nearly all of the proof's reception has involved canvassing the prospects for some form of verificationism.² Unfortunately, debates of this sort discount much of the philosophical import of the proof. In addition to its relevance for verificationism, Fitch's proof is also an argument for the existence of God, one at least as strong as the traditional demonstrations. Perhaps unlike other such proofs, Fitch's also operates as a key lemma in a proof that (if sound) establishes that God can't exist.

While the implications of Fitch's proof are thus very important for our understanding of key concepts in the philosophy of religion, they are also relevant to the proof's traditional reception. With these results, I am able to provide a principled motivation for Neil Tennant's recent defense of a restricted form of verificationism.³

1 F. Fitch, 'A Logical Analysis of Some Value Concepts,' *Journal of Symbolic Logic* 28 (1963) 135-42.

2 See the excellent overview of such responses in J.L. Kvanving, 'The Knowability Paradox and the Prospects for Anti-Realism,' *Nous* 29 (1995) 481-500.

3 Neil Tennant, *The Taming of the True* (Oxford: Oxford University Press 1997), ch. 8.

I A New Epistemic Paradox

Fitch's proof presupposes the validity of one transformation and four inference rules concerning knowledge and necessity. The transformation rule can be presented in this manner.

$$\begin{array}{l} \boxed{} \text{ intro.} \\ \quad | - P, \text{ therefore } | - \boxed{P} \end{array}$$

This rule says that if P is provable from no assumptions, then the necessity of P is also provable from no assumptions. Three of the other inference rules are:

$$\begin{array}{l} K \text{ elim.} \\ \quad \exists x KxP \quad | - P \\ K \wedge \text{ dist.} \\ \quad \exists x Kx(P \wedge Q) \quad | - \exists x KxP \wedge \exists x KxQ \\ \boxed{} \neg \text{ dist.} \\ \quad \boxed{\neg P} \quad | - \neg \diamond P \end{array}$$

The first of these says that if there exists someone who knows a claim, then that claim is true; the second that if a conjunction is known then each of its conjuncts is also known; and the third that anything that is necessarily not the case is also impossible. It is the next inference rule that has incited the most controversy.

$$\begin{array}{l} V.: \\ \quad P \quad | - \diamond \exists x KxP \end{array}$$

This says that P implies that it is possible to know that P . With these rules, we can present Fitch's proof in a Fitch style proof system.

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|--------------------------------------------------------------|-----------------------------------------------------------------|
| 1. $\exists x Kx(P \wedge \neg \exists y KyP)$ | assumption for \neg intro.
(<i>reductio ad absurdum</i>) |
| 2. $\exists x KxP \wedge \exists x Kx(\neg \exists y KyP)$ | 1 K \wedge dist. |
| 3. $\exists x KxP$ | 2 \wedge elim. |
| 4. $\exists x Kx(\neg \exists y KyP)$ | 2 \wedge elim. |
| 5. $\neg \exists y KyP$ | 4 K elim. |
| 6. $\neg \exists x KxP$ | 5 logical equivalence |
| 7. # | 3,6 \neg elim. |

8. $\neg\exists x Kx(P \wedge \neg\exists y KyP)$ 1-7 \neg intro.
 9. $[\]\neg\exists x Kx(P \wedge \neg\exists y KyP)$ 8 $[\]$ intro.
 10. $\neg\Diamond\exists x Kx(P \wedge \neg\exists y KyP)$ 9 $[\]\neg$ dist.
 11. $\mid P \wedge \neg\exists y KyP$ assumption for \neg intro.
 12. $\mid \Diamond\exists x Kx(P \wedge \neg\exists y KyP)$ 11 V.
 13. $\mid \#$ 10,12 \neg elim.
 14. $\neg(P \wedge \neg\exists y KyP)$ 11-13 \neg intro.
 15. $P \rightarrow \exists y KyP$ 14 (strictly classical)⁴ equivalence.

While this proof is deeply puzzling, it is not a paradox in the strict sense, where a contradiction is derived from no premises.

Interestingly though, Fitch's proof operates as a key lemma in such a paradox. Only one more inference rule is required.

$$K\neg dist.: \\ \exists x Kx\neg P \mid - \neg\exists x KxP$$

This rule merely says that if it is known that P is false then it is not known that P .⁵

Then, to derive a paradox we need merely consider the self-referring sentence, 'This sentence is not knowable.' Call this sentence Q . Given this meaning, the following inference is licensed for Q .

$$Q sub.: \\ P[Q] - \mid \mid - P[\neg\Diamond\exists x KxQ]$$

4 Intuitionists might reject the transition from line 13 to 14, as $P \rightarrow \exists y KyP$ only really follows from $\neg(P \wedge \neg\exists y KyP)$ with the help of a classical negation rule such as the law of excluded middle or double negation elimination. In T. Williamson, 'On the Paradox of Knowability,' *Mind* (1987) 256-61, the author suggests that this might be thought of as providing evidence for intuitionism, albeit not very much. The denial that any claim can be both true and unknown, as stated schematically in line 13, is problematic enough.

5 Note that a dialethist such as J.C. Beall might very well reject this assumption. For a dialethist response to Fitch's proof see J.C. Beall, 'Fitch's Proof, Verificationism, and the Knower Paradox,' *Australasian Journal of Philosophy* 78 (2000) 241-7.

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|---------------|-------------------|
| 3. TR | 1 T intro. |
| 4. # | 2,3 \neg elim. |
| 5. $\neg R$ | 1-4 \neg intro. |
| 6. T $\neg R$ | 5 T intro. |
| 7. $\neg TR$ | 6 T \neg dist. |
| 8. R | 7 R sub. |
| 9. # | 5,8 \neg elim. |

This is nothing more than a regimentation of the standard liar paradox.

For another example, consider the meta-linguistic reasoning involved in using Gödel's self-referring sentence that in effect says, 'I am not provable' to show that there can be no sound and complete axiomatization of number theory. The corresponding sub rule would be the following.

$$G \text{ sub.}: \\ P[G] - | \neg P[\neg \neg_N G]$$

Then, the assumption of completeness is that there exists an axiomatizable provability relation such that for all sentences P , either P is provable or refutable ($\forall P (|_{\neg_N} P \vee |_{\neg_N} \neg P)$); the assumption of consistency is that there does not exist a P such that P and its negation are provable ($\neg \exists P (|_{\neg_N} P \wedge |_{\neg_N} \neg P)$); and the assumption of the soundness of the provability relationship can be expressed by the inference from P 's provability to P ($\forall P (|_{\neg_N} P \rightarrow P)$). Then, the incompleteness argument can be regimented in the following manner.

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|-------------------------------------------------------------|------------------------------------------|
| 1. $\forall P (_{\neg_N} P \vee _{\neg_N} \neg P)$ | by the completeness of $ _{\neg_N}$ |
| 2. $\neg \exists P (_{\neg_N} P \wedge _{\neg_N} \neg P)$ | by the consistency of $ _{\neg_N}$ |
| 3. $\forall P (_{\neg_N} P \rightarrow P)$ | by the soundness of $ _{\neg_N}$ |
| 4. $ _{\neg_N} G \vee _{\neg_N} \neg G$ | 1 \forall elim. |
| 5. $ _{\neg_N} G$ | assumption for \vee elim. |
| 6. G | 3,5 \forall elim., \rightarrow elim. |
| 7. $\neg _{\neg_N} G$ | 6 G sub. |
| 8. # | 5,7 \neg elim. |

9. $ \neg \neg \neg G$	assumption for \vee elim.
10. $ \neg G$	3,9 \vee elim., \rightarrow elim.
11. $ \neg \neg \neg_N G$	10 G sub.
12. $ \neg_N G$	11 double negation elim.
13. $ \neg_N G \wedge \neg_N \neg G$	9,12 \wedge intro.
14. $ \exists P(\neg_N P \wedge \neg_N \neg P)$	13 \exists intro.
15. $ \#$	2,14 \neg elim
16. $\#$	4, 5-8, 9-15 \vee elim.

Thus does the Gödel sentence undermine hope of axiomatizing number theory.

I hope this is enough to provide evidence for the fact that Q sub. type rules are presupposed in standard, non-formal statements of non-trivial arguments involving self-reference. With Q sub. we can continue Fitch's proof to derive a genuine paradox.

16. $ Q$	assumption for \neg intro.
17. $ \neg \diamond \exists x KxQ$	16 Q sub.
18. $ \diamond \exists x KxQ$	16 V .
19. $ \#$	17,18 \neg elim.
20. $\neg Q$	16-18 \neg intro.
21. $\exists x Kx \neg Q$	15,20 \rightarrow elim. ⁸
22. $\neg \exists x Kx Q$	21 $K\neg$ dist.
23. $[] \neg \exists x KxQ$	22 $[]$ intro.
24. $\neg \diamond \exists x Kx Q$	23 $[] \neg$ dist.
25. $\neg \neg \diamond \exists x KxQ$	20 Q sub.
23. $\#$	24, 25 \neg elim.

8 This is O.K., since the conditional in 15 is schematic.

This is a genuine paradox. A contradiction is derived from no premises.

Prior to discussing the interesting ramifications of this argument, a potential objection must be noted. One might object that what I, in good Gentzenian fashion, have called ‘[] intro.’ is standardly referred to as the rule of necessitation, and that the rule of necessitation is only intended to apply to logical truths. Since both *Q* sub. and *V.* are not logical inferences, the rule of necessitation does not apply, and the proof is invalid.⁹

Such an argument can be quickly deflected, albeit it does raise questions about the status of *V.* in the argument. For the deflection, one need merely replace *Q* sub. and *V.*, respectively, with the axiom schemas: $[(P[Q] \rightarrow P[\neg\Diamond\exists x KxQ])]$ and $[(P \rightarrow \Diamond\exists x KxP)]$, and exploit the fact that $[(P \rightarrow Q)]$ entails $[(P \rightarrow [Q])]$. Then, the application of necessitation in the proof to paradox can be made to apply to a conditional with the parts of the axiom schemas bound by the necessity box as the antecedent.¹⁰ Instances of the schema corresponding to *Q* sub. are clearly necessarily true, since *Q* sub. is true in virtue of the meaning of *Q*.

However, one might argue that instances of the axiom schema corresponding to *V.* are not necessarily true. In fact, for the anti-Dummettian there would be instances that are actually false. Thus, viewing the argument as a paradox, and not merely as a refutation of Dummettian anti-realism, crucially depends upon one’s view of the status of verificationism.

One should not think that Fitch’s proof and our demonstration are merely evidence against positivistic verificationism, as encoded by the inference *V.* For, a central tenant of positivist verifiability involves construing the kind of knowability mentioned in the principle *V.* as limited by human epistemic capacities. But none of our three inference rules regarding knowledge above require knowability to be so limited or, for that matter, tied to a human being’s capacity to know in *any* manner.

In another context, Michael Dummett states this point well, arguing that he (the neo-positivist) and his opponent share a commitment to verificationism.

In this way, we try to convince ourselves that our understanding of what it is for undecidable sentences to be true consists in our grasp of what it would be to be able to use such sentences to give direct reports of observation. We cannot do this; but we know just what powers a superhuman observer would have to have in order to

9 I owe this objection to an anonymous reviewer.

10 See the appendix for a restatement of the proof along these lines.

be able to do it — a hypothetical being for whom the sentences in question would *not* be undecidable. And we tacitly suppose that it is in our conception of the powers which such a superhuman observer would have to have, and how he would determine the truth-values of the sentences, that our understanding of their truth-conditions consists. This line of thought is related to a second regulative principle governing the notion of truth: If a statement is true, it must be in principle possible to know that it is true. This principle is closely connected with the first one [that if a statement is true, then there must be something which makes it true]: for, if it were in principle impossible to know the truth of some true statement, how could there be anything which *made* that statement true?¹¹

Thus does Dummett argue for a constitutive connection between the concepts of a sentence being made true and that of somebody knowing that it is true. This should *not* be confused with the Dummettian challenge to the metaphysical realist.

Dummett explicitly argues that many debates concerning realism should be construed in terms of how much idealization is allowed in the principle that every true sentence is knowable. The realist interprets 'it is possible to know *P*,' essentially as 'if God does (were to) exist, then God does (would) know that *P*,'¹² while the Dummettian anti-realist insists that the sense of knowability must reflect human limitations. Recent discussions of verificationism amplify Dummett's contention.¹³

Perhaps most important, if Dummett is correct, then V. is a principle that both neo-positivists and non-positivists should accept. Since V. (as well as the other inferences concerning knowledge and possibility) makes no concession to Dummettian neo-positivism, the strategy of construing Fitch's proof and the above paradox as merely a refutation of positivistic verificationism fails. Rather, our paradox is a genuine paradox, a contradiction seemingly coming out of nowhere.¹⁴

11 Michael Dummett, 'What is a Theory of Meaning (II),' in *The Seas of Language*, M. Dummett (Oxford: Oxford University Press 1993), 61.

12 Nothing in Dummett's point requires the existence of a God, as Dummett's realist only need think that it is *possible* that God exists, and that this being's knowledge is relevant to our understanding of language. Thus, our discussion is orthogonal to Plantiga's argument's for theistic anti-realism in A. Plantiga, 'How to Be an Anti-Realist,' *Proceedings of the American Philosophical Association* (1982): 47-70.

13 For example, see C. Misak, *Verificationism*, (London: Routledge 1995), N. Jardine, *The Fortunes of Inquiry*, (Oxford: Oxford University Press 1986), and M. Hand and J.L. Kvanvig, 'Tennant on Knowability,' *Australasian Journal of Philosophy* 77 (1999) 422-8.

14 One should note that the inference K elim. and the conclusion of Fitch's proof together establish that the 'is known' predicate is coextensive with the truth predicate. Thus, we could have let *Q* just be '*Q* is not known' and presented a notational

This is in contrast to recent formulations of traditional epistemic paradoxes, which make more substantive assumptions about knowability. For example, Richard Montague assumes that his knowability predicate is closed under deduction.¹⁵ More recently, J.C. Beall assumes that if we prove a claim then it is known, even when the proof in question is part of a paradox.¹⁶ Montague's notion of knowability is superhuman, while Beall's of knowledge perhaps all too human. That Fitch's notion isn't committed in these regards renders the self-referential paradox in which Fitch's proof occurs all the more startling.

II Some Consequences Concerning the Existence of God

Dummett's implicit appeal to a 'God's eye view' illustrates a fascinating aspect of Fitch's proof that has not been noticed. Fitch's proof itself can be used to produce a new inductive argument for God's existence.

1. Consider any true sentence P .
2. Since God is omniscient, if God were to exist, then God would know that P .¹⁷
3. It is at least possible that God exists.
4. But then, from lines 2 and 3, it is at least possible that it is known that P .
5. Thus, from the preceding lines we have that if a sentence is true then it is possible that it is known ($P \mid - \Diamond \exists x KxP$).

variant of the standard liar paradox! Thus, it is the following philosophical discussion that justifies our use of knowability.

15 R. Montague, 'Syntactical Treatments of Modality with Corollaries on Reflexion Principles and Finite Axiomatizability,' *Acta Philosophica Fennica* 16 (1963) 153-67, and R. Montague and D. Kaplan, 'A Paradox Regained,' *Notre Dame Journal of Formal Logic* 1 (1960) 79-90.

16 Beall, *ibid.*

17 On the assumption that P is true in the closest possible world in which God exists. This is no very big concession though, as the inference only fails then for any true sentences inconsistent with God's existence. Were the domain of quantification restricted to such sentences, the proof is clearly still inductively valid.

6. But then from Fitch's proof and the preceding line we have that if a sentence is true, then it is known to be true.
7. Since the claim that all truths are known is what you would expect if God existed (in this case, they would be known by God), line 6 is very good inductive evidence for the existence of God.

The similarities of this argument to Anselm's traditional ontological argument are clear.¹⁸ Both arguments reason from the claim that God is conceived in a certain way to the conclusion that God exists. In Anselm's case it is the conception of God as 'that than which no greater can be conceived' that does the dialectical work. Our argument trades on God's purported omniscience.

The comparison with Anselm's proof illustrates the importance of our epistemic paradox as well. One might suspect that Anselm's proof actually shows us that the notion of a being which is 'that than which no greater can be conceived' is an incoherent notion. In a similar manner our paradox seems to show that the concept of omniscience is incoherent. Since the mere possibility of God existing motivates the validity of our verificationist inference (V.), the paradox arrived at by using the inference motivates the impossibility of God's existing. If Fitch's proof is a very good inductive argument for the truth of God's existence, our paradox is a very good deductive argument for the impossibility of God's existence.

III Some Consequences of the Consequences

The theist might be tempted to devise a solution to our paradox that leaves Fitch's proof unscathed. One that immediately suggests itself is semantic ascent. Perhaps the predicate 'is known' applies to sentences one or more levels down in a hierarchy of metalanguages.¹⁹ Then, if a

18 The theological relevance of Fitch's proof was first noted in J. MacIntosh, 'Fitch's Fatives,' *Analysis* 44 (1984) 145-58, and J. MacIntosh, 'Theological Question-Begging,' *Dialogue* 30 (1991) 531-47. MacIntosh notes the similarity of Fitch's proof and the ontological argument. However, MacIntosh does not consider epistemic notions, nor does he use Fitch's proof in an argument against God's existence.

19 Montague (*ibid.*) took this to be the moral of his epistemic paradox. This might provide independent motivation for its employment in relation to this paradox. On the other hand, to the extent that one does not find it well motivated here, one might begin to question it in relation to Montague's paradox.

sentence is prohibited from containing a sentence higher up in the hierarchy, our liar-type sentence 'I am not knowable' is ill-formed. Then there would simply not be a sentence Q such that Q sub. is a valid rule of inference. Paradox blocked!

This is a familiar maneuver to block self-referential paradoxes.²⁰ I shall neither defend nor criticize it. Our concern is solely whether semantic ascent blocks our paradox while leaving Fitch's proof unscathed. One would think that it succeeds beautifully, as Fitch's proof does not involve self-reference. However, this makes it doubly strange that semantic ascent works at least as well as part of a strategy for blocking Fitch's proof.

The proof begins by considering the sentence $\exists xKx(P \wedge \neg\exists yKyP)$. On the assumption that the K predicate applies to sentences at lower levels, we have $\exists xK_3x(P_1 \wedge \neg\exists yK_2yP_1)$ ²¹ instead. But then the beginning of Fitch's proof becomes the following.

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|----------------------------------------------------------------------|-----------------------------------------------------------------|
| 1. $\exists xK_3x(P_1 \wedge \neg\exists yK_2yP_1)$ | assumption for \neg intro.
(<i>reductio ad absurdum</i>) |
| 2. $\exists xK_3xP_1 \wedge \exists x K_3x(\neg\exists y K_2yP_1)$ | 1 K dist. |
| 3. $\exists xK_3xP_1$ | 2 \wedge elim. |
| 4. $\exists xK_3x(\neg\exists yK_2yP_1)$ | 2 \wedge elim. |
| 5. $\neg\exists yK_2yP_1$ | 4 K elim. |
| 6. $\neg\exists x K_2xP_1$ | 5 logical equivalence |
| 7. # | 3,6 \neg elim. |

But, one could argue that there is no contradiction in saying that P is known in the second meta-language ($\exists xK_3xP_1$) and not known in the first ($\neg\exists xK_2xP_1$). Indeed, if meta-languages are genuinely more expressive than object languages, this is not at all surprising. I conclude that theist

20 See A. Tarski, 'The Semantic Conception of Truth and the Foundations of Semantics,' *Philosophy and Phenomenological Research* 4 (1944): 341-76.

21 Assuming that P is level one clearly involves no loss of generality. Also, one might implement semantic ascent slightly differently. Perhaps K only applies to sentences one level down. Perhaps only sentences of the same level can be conjoined. In these cases the premise assumed for *reductio* would be $\exists xK_3x(P_2 \wedge \neg\exists yK_2yP_1)$ (P_2 being the canonical metalinguistic translate of P_1). In these cases, our point would still hold. Line 6 would still not be a contradiction.

should not use this strategy to save Fitch's proof while blocking our paradox.²²

Another strategy is suggested by analogy. If 'K' is understood as 'is done by someone,' then our paradox is extraordinarily similar to the traditional 'paradox of the stone' concerning God's omnipotence. The paradox of the stone considers the question of whether God could create a stone he couldn't lift. But then it's possible that a stone exists which God couldn't lift, so God isn't omnipotent after all (there being a possible act that God couldn't do). So it seems that God can't create an unliftable stone. But then again it follows that God isn't omnipotent after all.

The standard response²³ is that we should not define omnipotence as the possibility of doing anything whatsoever. Rather we should say that an omnipotent being is able to do anything that it is consistent to suppose done. To be more precise, where ' $\exists xDxP$ ' means ' P is done' the possible existence of an omnipotent being might have tempted us to accept the following inference rule.

$$\mid - \diamond \exists xDxP$$

This logically entails the following inference rule.

$$D.: \\ P \mid - \diamond \exists xDxP$$

This says that if P is true, then it is possible that P is brought about, or done, by someone. Again, the possible existence of an omnipotent being seems to guarantee this. However, reflection on the paradox of the stone leads us to restrict D by replacing it with the following transformation rule.

22 I assume, perhaps wrongly, that the burden is on the theist to show that $\exists xK_3xP_1$ and $\neg \exists xK_2xP_1$ do contradict each other. Perhaps the burden of proof is upon the theist's enemy to show how $\exists xK_3xP_1$ and $\neg \exists xK_2xP_1$ can be consistent. Unfortunately for the anti-theist though, success in this endeavor will almost certainly undermine the use of semantic ascent to block the paradox too. This strikes me as an extremely important issue for defenders of meta-linguistic resolutions of self-referential paradoxes, and one interesting in its own right for those who find such approaches ill-motivated. Such a discussion does not lie within the scope of this paper, since I am concerned with defending a different strategy for Fitch's proof and the new paradox.

23 G. Mavrodes, 'Some Puzzles Concerning Omnipotence,' *The Philosophical Review* 72 (1963) 221-3.

$$D.' : \\ \neg(\exists x DxP \mid - \perp), \text{ therefore } P \mid - \diamond \exists x DxP$$

This says that if the assumption that P is done does not entail a contradiction, then the truth of P entails the possibility of P being done or brought about that. For once we have redefined omnipotence in light of the paradox of the stone, the possible existence of an omnipotent being at best motivates $D.'$

So God can only do what it is consistent to suppose done. An analogous restriction on $V.$ would give us the following.

$$V.' : \\ \neg(\exists x KxP \mid - \perp), \text{ therefore } P \mid - \diamond \exists x KxP$$

Eureka! Just as $D.'$ blocks the paradox of the stone, so $V.'$ blocks our paradox. Consider the following bit of the proof of our paradox.

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|----------------------------------------|------------------------------|
| 16. $\mid Q$ | assumption for \neg intro. |
| 17. $\mid \neg \diamond \exists x KxQ$ | 16 Q sub. |
| 18. $\mid \diamond \exists x KxQ$ | 16 $V.$ |

It is easy to show that line 18 does not follow from line 16 with the restricted rule $V.'$ This requires use of only one more inference rule.

$$\diamond \text{ intro.} : \\ P \mid - \diamond P$$

This rule merely says that if a sentence is true, then it is possible that it is true. Now consider the following proof.

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|----------------------------------|------------------------------|
| 1. $\exists x KxQ$ | |
| 2. Q | 1 K elim. |
| 3. $\neg \diamond \exists x KxQ$ | 2 Q sub. |
| 4. $\diamond \exists x KxQ$ | 1 \diamond intro. |
| 5. # | \neg intro. Contradiction! |

Eureka again, since the assumption that Q is known leads to absurdity, the inference rule $V.'$ can not be applied to yield line 18 above. Our paradox is blocked.

Unfortunately for the theist, this solution to our paradox also blocks Fitch's proof. The key question is whether $V.'$ can operate as $V.$ did in the original proof, repeated here.

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|---------------------------------------------------------|------------------------------|
| 11. $P \wedge \neg\exists yKyP$ | assumption for \neg intro. |
| 12. $\diamond\exists x Kx(P \wedge \neg\exists yKyP)$ | 11 $V.$ |

It is clear that this inference is not valid with $V.'$ for the first six lines of our presentation of Fitch's proof show that $\exists xKx(P \wedge \neg\exists yKyP)$ proves absurdity.

- | | |
|-----------------------------------------------------------|-----------------------|
| 1. $\exists x Kx(P \wedge \neg\exists yKyP)$ | |
| 2. $\exists x KxP \wedge \exists x Kx(\neg\exists y KyP)$ | 1 $K \wedge$ dist. |
| 3. $\exists x KxP$ | 2 \wedge elim. |
| 4. $\exists x Kx(\neg\exists y KyP)$ | 2 \wedge elim. |
| 5. $\neg\exists y KyP$ | 4 K elim. |
| 6. $\neg\exists x KxP$ | 5 logical equivalence |
| 7. # | 3,6 \neg elim. |

Once again, an attempt to block the paradox succeeds in blocking Fitch's proof as well.

That Fitch's proof fails when $V.$ is replaced by $V.'$ has been shown in Neil Tennant's recent book *The Taming of the True*, where Fitch's proof leads Tennant to use the restricted principle throughout the book. If one considers this restriction when just thinking of Fitch's proof, it is likely to strike one as it strikes Timothy Williamson, Michael Hand, and Jonathan L. Kvanvig,²⁴ that is, as being horribly and irredeemably *ad hoc*. If this reaction is justified, then it follows from our reflections that the traditional solution to the paradox of the stone is also suspect!

On the other hand, one person's *modus tollens* is another's *modus ponens*. One might think that since the traditional solution to the paradox of the stone is perfectly reasonable, so is the analogous solution to our paradox of knowability. But then Tennant's solution to Fitch's proof has been shown to be well motivated.

24 See Timothy Williamson, 'Tennant on Knowable Truth,' *Ratio* 13 (2000) 99-114 and Hand and Kvanvig, *ibid*.

IV The Charge of *Ad Hocness*

The only way out of this impasse is to adduce independent arguments for or against the claim that Tennant's solution is *ad hoc*. Tennant himself seems to be aware of this. Where a proposition is Cartesian if and only if it is consistent to suppose it known, Tennant makes the following argument.

Does our restriction [of V. to V.'] render the knowability principle thereby expressed toothless? Does the restriction to Cartesian truths secure the truth of the knowability principle by sheer stipulation? Does it amount to claiming that the principle holds except where it doesn't? Not at all. Think of all of the Cartesian propositions of mathematics and empirical science, propositions that involve no mention, within them, of epistemic notions. To claim that every such truth is in principle knowable is still to forswear metaphysical realism.²⁵

Tennant's argument is that the restriction is not *ad hoc* in virtue of the fact that V.' is not trivial, since the question of whether it is true still marks a significant dispute between the realist and anti-realist. In 'Tennant on Knowability,' Hand and Kvanvig correctly note that Tennant's argument is a *non sequitur*.

Tennant holds that "to claim that every [ordinary truth of mathematics and science] is in principle knowable is still to forswear metaphysical realism." This means that realism is incompatible with the hypothesis that there could be an omniscient individual. It is not.²⁶

This is nothing more than Michael Dummett's point, discussed above, to the effect that the real metaphysical debate concerns how to construe the epistemic modals in the statement of verificationism. Where ' $\exists_{\text{god}}xKxP$ ' means that P is knowable to God, and ' $\exists_{\text{us}}xKxP$ ' means that P is knowable to us, Dummett's realist and anti-realist can be presented as holding, respectively, the following two positions.

RV.:
 $P \mid - \Diamond \exists_{\text{god}}xKxP$

ARV.:
 $P \mid - \Diamond \exists_{\text{us}}xKxP$

25 Tennant, 275.

26 Hand and Kvanvig, 427.

If we accepted Tennant's solution, the two positions could then be presented in the following way.

RV.':

$\neg(\exists_{\text{god}}xKxP \mid - \perp)$, therefore $P \mid - \Diamond_{\text{god}}xKxP$

ARV.':

$\neg(\exists_{\text{us}}xKxP \mid - \perp)$, therefore $P \mid - \Diamond_{\text{us}}xKxP$

As Hand and Kvanvig argue, it is thus very clear that the *non-disambiguated* principle V.' does not decide any issues concerning Dummettian realism and anti-realism.²⁷ But then Tennant's attempt to show that his solution is principled fails.

What is quite strange is the fact that, earlier in their discussion, Hand and Kvanvig note a way in which Tennant's position is non-trivial. Discussing Tennant's restriction, they write,

It is no less unprincipled to say that p is necessarily knowable except when the assumption that p is known is inconsistent than to say that p is necessarily knowable except when it is not.²⁸

My point is that Tennant is *not* saying that if it is possible that P is known, then it is possible that P is known (which nobody could argue about). What Tennant is saying is that if it is logically possible that P is known (and P is true), then it is possible in a weaker modality that P is known.²⁹ This must be what Tennant means, given that logical possibility is the strongest modality in the sense that many things are logically possible

27 Note that there is also the principle even more anti-realist than ARV.' to the effect that if it is consistent to suppose God knows P (and P is true), then it is possible for us to know P . The moral of this, and Hand and Kvanvig's discussion (though clearly not one they endorse) is that the principle that all true claims are knowable might be completely irrelevant to the substantive debates between Dummettian anti-realists and their opponents! It seems that all along Dummettians and their opponents should have been arguing over the principle that if a claim is knowable at all then it is knowable to creatures like us. Fitch's proof does not undermine this principle. Whether this principle is felicitous for arguments for intuitionistic revision is, I think, an extremely important open question.

28 *ibid.*, 424.

29 Depending upon one's philosophy of logic, one might object that failure to derive a contradiction, or consistency, is not the same as logical possibility. But the principle that if it is consistent to assume that P is known (and P is true), then it is possible in some weaker modality for P to be known is still non-trivial. How non-trivial depends upon the characterization of the weaker modality.

and not possible in other modalities, such as physical or metaphysical. Whether or not this engages with Dummett's dialectic, is an *extremely* non-trivial principle, how extreme depending upon the extent of the weaker modality's weakness. For example, the restricted principle does *not* hold of the concept of doing with respect to logical and physical possibility. It could be logically possible yet physically impossible that God brought some existing state of affairs about. This shows that if the fact that the resultant position is non-trivial is sufficient for not being *ad hoc*, then Tennant's position is not *ad hoc*.

To grasp how central the different strengths of modality are here, consider a possible counterargument to Tennant.³⁰ One might argue that Tennant's solution is not a solution at all, since his proposed principle is vacuously true. Consider the following proof

- | | |
|-------------------------------------------------------------------------------------|----------------------------------------------|
| 1. $\neg\Diamond\exists xKxP \vee \Diamond\exists xKxP$ | tautology |
| 2. $\Box\neg\exists xKxP \vee (P \rightarrow \Diamond\exists x KxP)$ | 1 ordinary modal and
sentential logic |
| 3. $(\exists xKxP \mid - \#) \vee (P \rightarrow \Diamond\exists x KxP)$ | 2 $\Box\neg P \mid - \Box(P \rightarrow \#)$ |
| 4. $\neg(\exists xKxP \mid - \#) \rightarrow (P \rightarrow \Diamond\exists x KxP)$ | 3 by sentential logic |

So, Tennant's 'solution' is a pseudo-solution, since it follows by ordinary logic from a tautology. This, line of reasoning is fallacious though, as deriving line 3 involves equating necessity with provability! The inference in question, $\Box\neg P \mid - \Box(P \rightarrow \#)$, really would only give you the following.

- | | |
|------------------------------------------------------------------------------------|-------------------------------------------------|
| 3'. $\Box(\exists xKxP \rightarrow \#) \vee (P \rightarrow \Diamond\exists x KxQ)$ | 1, $\Box\neg P \mid - \Box(P \rightarrow \#)$. |
|------------------------------------------------------------------------------------|-------------------------------------------------|

But then one could only infer line 3 above if one held that $\Box(P \rightarrow \#)$ entails $\mid - (P \rightarrow \#)$. This would involve continuing the proof in this manner.

- | | |
|---------------------------------------------------------------------------------------|--------------------------------------------------------------------------|
| 4'. $\mid - (\exists xKxP \rightarrow \#) \vee (P \rightarrow \Diamond\exists x KxQ)$ | 1, $\Box(P \rightarrow \#)$, therefore
$\mid - (P \rightarrow \#)$. |
|---------------------------------------------------------------------------------------|--------------------------------------------------------------------------|

Only then could one apply the deduction theorem to get line 4 of the above proof. However, as I have already shown, the non-triviality of Tennant's solution crucially rests on distinguishing the kind of necessity occurring in the verificationist principle from logical necessity. Thus, for

30 I owe this counter-argument to an anonymous reviewer.

Tennant, $\Box(P \rightarrow \#)$, therefore $\Box \neg (P \rightarrow \#)$ is *not* a valid inference. If all necessity were logical necessity (and hence all possibility, logical possibility) of course Tennant's restricted principle would be trivial. As it would merely say that if it is possible that P is known, then if P is true, then it is possible that P is known. This would be a completely uninformative tautology. But Tennant never defended such a principle. Rather, the principle says that if it is logically possible that P is known, then if P is true, then it is possible in some weaker modality that P is known.³¹

Things are apparently still bad for Tennant though. Not only do Hand and Kvanvig attempt to undermine Tennant's defense of his restricted principle as non-trivial, they produce positive arguments to the conclusion that Tennant's principle is *ad hoc*. One such argument is the following argument by analogy.

... consider Russell's paradox, which threatens the claim that any grammatically predicative expression defines a set. We can apply Tennant's strategy and avoid the paradox simply by saying that any such expression defines a set except when the assumption that it does so yields a contradiction. Such an approach to Russell's paradox is clearly *ad hoc*.³²

Clearly, if Tennant's solution were analogous to such a 'solution' to Russell's paradox, it would be inadequate. However, this is not the case. Where 'GPE[x]' means that the x occurs free in a grammatically predicative expression, and 'S x ' means that x names a set, the cause of Russell's paradox can be given in the following manner.

FFA- GPE[x] \mid - S x

Where FFA[y] is what you get when y is uniformly substituted for x in FFA. Hand and Kvanvig's projected solution is the following.

$\neg(\text{FFA}[y] \mid \neg) \wedge \text{GPE}[y]$, therefore S y

The analog 'solution' to Fitch's proof would thus be the following

V.: $P \mid \neg \Diamond \exists KxP$

'Solution' $\neg(\text{V.}[Q] \mid \neg) \wedge Q$, therefore $\Diamond \exists KxQ$

31 Much of Tennant's discussion is relevant to characterizing this form of possibility. See for example, Tennant, 153.

32 Hand and Kvanvig, 426.

If this is what Tennant proposed, Hand and Kvanvig would be right in asserting that Tennant's solution is, 'as if the anti-realist were to respond to counterexamples by asserting that anti-realism holds for all propositions except counterexamples.'³³ However, it should be clear that Tennant does *not* offer the above 'solution.' It *would* be unprincipled if he argued that 'the principle holds for a claim *P*, except when assuming it does hold for *P* leads to a contradiction.' But Tennant doesn't assert that; he says, 'the principle holds for *P*, except when assuming *P* is known leads to a contradiction.'³⁴

To be fair, Hand and Kvanvig only say that it is 'as if' Tennant had offered the above 'solution,' but their argument for this claim also relies on a false analogy. In the case of Russell's paradox, solving it involved developing modern set theory. Here we did get an 'account of the nature of sets that precludes the problematic ones,'³⁵ and so much more. Hand and Kvanvig charge a non *ad hoc* solution to Fitch's proof with similarly illuminating the key concepts in the proof. They write

What is needed is *an account of the nature of truth* that prevents problematic ones from making trouble for anti-realism. Tennant's proposal simply does not do that.³⁶

It would, no doubt, be wonderful if Fitch's proof were even a fraction as important as Russell's paradox in this way. But is there any reason to think so? Tennant should just respond that epistemic logic is already well understood in a way that the foundations of mathematics weren't at the time of Russell's paradox. That is, Tennant should be thought of as exploiting our understanding of consistency in epistemic and modal logics to solve the paradox. Nothing of this sort could have taken place during the time of Russell's paradox. The analogy fails.

This is not to deny that shedding light upon possibility and knowledge is a goodmaking feature of solutions to Fitch's proof. The problem with Hand and Kvanvig's critique of Tennant is that they assume that it is a necessary condition on solutions that they so deepen our understanding.

33 *ibid.*, p. 425.

34 My earlier discussion of self-referential paradoxes shows that a better example *might* have involved an analog of Tennant's solution when considering the liar paradox. I don't, however, think this would work for precisely the reason that Tennant's solution is non-trivial, the role of different kinds of modality, which don't occur at least in standard versions of the liar paradox.

35 *ibid.*, 426.

36 *ibid.*

What if Zermelo-Fraenkel set theory had been developed in ignorance of Russell's paradox? Would it not then seem *ad hoc* to restrict the intuitive notion of set to members of the ZF hierarchy? Conversely, suppose people had realized the similarity between our epistemic paradox and the paradox of the stone long ago, and developed epistemic logic in a way to make sense of Tennant's restriction. This situation is exactly analogous to the development of Zermelo-Fraenkel set theory. In that possible world nobody would say that Tennant's solution was *ad hoc*. Nor should we say it's *ad hoc* in the actual world. Rather, we should say that Tennant's solution seems *ad hoc* because we already have a rigorous body of knowledge about how knowledge, possibility, and consistency interact.

Thus, we should opt for *modus ponens* rather than *modus tollens*. Since the traditional solution to the paradox of the stone is perfectly reasonable, so is the analogous solution to our paradox of knowability. The non-triviality of Tennant's restricted principle as well as the failure of Hand and Kvanvig's arguments show the *modus tollens* reaction (as goes Tennant's view, so goes the solution to the paradox of the stone) to be without merit. I conclude that Tennant's solution to Fitch's proof and the traditional solution to the paradox of the stone are not *ad hoc*.³⁷

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37 I must thank Roy Cook. This paper began as a joint paper with him that used the self-referential paradox to defend a meta-linguistic solution to Fitch's proof (Roy proved a fixed point theorem to get the sentence that says 'I am not knowable' in the appropriate kind of language, and helped interpret the levels such that Fitch's proof was intuitively invalid). While the strategy seemed promising for Fitch's proof, an ever increasing Priestian disenchantment with semantic ascent as a solution to self-referential paradoxes in general (and the liar paradox in particular) led us to abandon the project. In this vein, one must admit that showing Tennant's solution to be free from *ad hocness* is not the same as showing it true. In particular, one would need to see how J.C. Beall's dialethist solution to Fitch's proof (*ibid.*) applies to the paradox of the stone and interacts with other facets of Dummett's program, such as Dummett's arguments for intuitionism. I hope that it is clear how the above demonstrations motivate such a project. I am also grateful to an anonymous reviewer, Andrew Arlig, Emily Beck Cogburn, Stewart Shapiro, and Mark Silcox for helpful suggestions.

Appendix: Derivation of Paradox Using the Rule of Necessitation

- | | |
|-------------------------------------------------------|-------------------------|
| 16. (Q → ¬◇∃x KxQ) | assumption for → intro. |
| 17. (Q → ◇∃x KxQ) | assumption for → intro. |
| 18. Q | assumption for ¬ intro. |
| 19. ¬◇∃x KxQ | 16,18 → elim. |
| 20. ◇∃x KxQ | 17,18 → elim. |
| 21. # | 19,20 ¬ elim. |
| 22. ¬Q | 18-21 ¬ intro. |
| 23. ∃x Kx ¬Q | 15,22 → elim. |
| 24. ¬∃x Kx Q | 23 K¬ dist. |
| 25. (Q → ◇∃x KxQ) → ¬∃x Kx Q | 17-24 → intro. |
| 26. (Q → ¬◇∃x KxQ) → ((Q → ◇∃x KxQ) → ¬∃x Kx Q) | 16-25 → intro. |
| 27. []((Q → ¬◇∃x KxQ) → ((Q → ◇∃x KxQ) → ¬∃x Kx Q)) | 26 necessitation |
| 28. []((Q → ¬◇∃x KxQ) → []((Q → ◇∃x KxQ) → ¬∃x Kx Q)) | 27 □→ dist. |
| 29. [](Q → ¬◇∃x KxQ) | axiom schema for Q. |
| 30. []((Q → ◇∃x KxQ) → ¬∃x Kx Q) | 28,29 → elim. |
| 31. []((Q → ◇∃x KxQ) → []¬∃x Kx Q) | 30 []→ dist. |
| 32. [](Q → ◇∃x KxQ) | axiom schema for V. |
| 33. []¬∃x Kx Q | 31,32 → elim. |
| 34. ¬◇∃x Kx Q | 33 []¬ dist. |
| 35. (¬Q → ¬¬◇∃x KxQ) | assumption for → intro. |
| 36. (Q → ◇∃x KxQ) | assumption for → intro. |
| 37. (Q → ¬◇∃x KxQ) | assumption for → intro. |

38. $| | | | Q$ assumption for \neg intro.
39. $| | | | \neg\forall x KxQ$ 37,38 \rightarrow elim.
40. $| | | | \forall\exists x KxQ$ 36,38 \rightarrow elim.
41. $| | | | \#$ 39,40 \neg elim.
42. $| | | \neg Q$ 38-41 \neg intro.
43. $| | | \neg\neg\forall\exists x KxQ$ 35,42 \rightarrow elim.
44. $| | (Q \rightarrow \neg\forall\exists x KxQ) \rightarrow \neg\neg\forall\exists x KxQ$
37-43 \rightarrow intro.
45. $| (Q \rightarrow \forall\exists x KxQ) \rightarrow ((Q \rightarrow \neg\forall\exists x KxQ) \rightarrow \neg\neg\forall\exists x KxQ)$
36-44 \rightarrow intro.
46. $(\neg Q \rightarrow \neg\neg\forall\exists x KxQ) \rightarrow ((Q \rightarrow \forall\exists x KxQ) \rightarrow$
 $((Q \rightarrow \neg\forall\exists x KxQ) \rightarrow \neg\neg\forall\exists x KxQ))$ 35-45 \rightarrow intro.
47. $\Box((\neg Q \rightarrow \neg\neg\forall\exists x KxQ) \rightarrow ((Q \rightarrow \forall\exists x KxQ) \rightarrow$
 $((Q \rightarrow \neg\forall\exists x KxQ) \rightarrow \neg\neg\forall\exists x KxQ)))$ 46 rule of necessitation
48. $\Box(\neg Q \rightarrow \neg\neg\forall\exists x KxQ) \rightarrow \Box((Q \rightarrow \forall\exists x KxQ) \rightarrow$
 $((Q \rightarrow \neg\forall\exists x KxQ) \rightarrow \neg\neg\forall\exists x KxQ))$ 47 $\Box \rightarrow$ dist.
49. $\Box(\neg Q \rightarrow \neg\neg\forall\exists x KxQ)$ axiom schema for Q .
50. $\Box((Q \rightarrow \forall\exists x KxQ) \rightarrow ((Q \rightarrow \neg\forall\exists x KxQ) \rightarrow \neg\neg\forall\exists x KxQ))$
48,49 \rightarrow elim.
51. $\Box(Q \rightarrow \forall\exists x KxQ) \rightarrow \Box((Q \rightarrow \neg\forall\exists x KxQ) \rightarrow \neg\neg\forall\exists x KxQ)$
50 $\Box \rightarrow$ dist.
52. $\Box(Q \rightarrow \forall\exists x KxQ)$ axiom schema for V .
53. $\Box((Q \rightarrow \neg\forall\exists x KxQ) \rightarrow \neg\neg\forall\exists x KxQ)$ 51,52 \rightarrow elim.
54. $\Box(Q \rightarrow \neg\forall\exists x KxQ) \rightarrow \Box\neg\neg\forall\exists x KxQ$ 53 $\Box \rightarrow$ dist.
55. $\Box(Q \rightarrow \neg\forall\exists x KxQ)$ axiom schema for Q .
56. $\Box\neg\neg\forall\exists x KxQ$ 54,55 \rightarrow elim.
57. $\neg\neg\forall\exists x KxQ$ 56 \Box elim.
58. $\#$ 34,57 \neg elim.